2. linear in y, not separable
not exact

6. not linear, not separable,
exact

8. linear in y, separable,
not exact

22. $\underbrace{(ye^{xy} - \frac{1}{y})}_{M}dx + \underbrace{(xe^{xy} + \frac{x}{y^2})}_{N}dy = 0$

$\frac{\partial M}{\partial y} = e^{xy} + xye^{xy} + \frac{1}{y^2}$

$= \frac{\partial N}{\partial x}$

$\therefore$ It is an exact eq

$\frac{\partial F}{\partial x} = ye^{xy} - \frac{1}{y}$

$\therefore F(x,y) = e^{xy} - \frac{x}{y} + g(y)$

$\frac{\partial F}{\partial y} = xe^{xy} + \frac{x}{y^2} + g'(y)$

$\equiv xe^{xy} + \frac{x}{y^2}$

$\therefore g'(y) = 0 \rightarrow g(y) = C$

$\therefore F(x,y) = e^{xy} - \frac{x}{y} + C$

$\therefore e^{xy} - \frac{x}{y} = C$

$y(1) = 1 \rightarrow e - 1 = C$

$\therefore \boxed{e^{xy} - \frac{x}{y} = e - 1}$

25. $\underbrace{(y^2 \sin x)}_{M}dx + \underbrace{(\frac{1}{x} - \frac{y}{x})}_{N}dy = 0$

$\left.\begin{aligned} My &= 2y\sin x \\ Nx &= -\frac{1}{x^2} + \frac{y}{x^2} \end{aligned}\right\}$ not exact!

But the eq is separable.

$(x\sin x)dx + (\frac{1}{y^2} - \frac{1}{y})dy = 0$

$\therefore \int(\frac{1}{y} - \frac{1}{y^2})dy = \int(x\sin x)dx$

$\therefore \ln|y| + \frac{1}{y} = -x\cos x + \int\cos x\,dx$

$= -x\cos x + \sin x + C$

Since $y(\pi) = 1 > 0$, we choose

$\ln y + \frac{1}{y} = -x\cos x + \sin x + C$

$0 + 1 = -\pi(-1) + 0 + C$

$\therefore C = 1 - \pi$

$\therefore \underline{\ln y + \frac{1}{y} = -x\cos x + \sin x + 1 - \pi}$

or $\underline{\sin x - x\cos x = \ln y + \frac{1}{y} + \pi - 1}$