LAB #6 - Mathematics 140

Probability, Confidence Intervals, Hypothesis Testing, and necessary sample size

1. The effect of anesthetic on the flow of aqueous humour (a fluid in the eye) was investigated in the paper "A Method for Near-Continuous Determination of Aqueous Humour Flow: Effects of Anesthetics, Temperature, and Indomethacin"' *Exper. Eye Research. (1984): 435-53*. Summary quantities for aqueous flow rate (μl/min) observed under three different anesthetics are given.

Anesthetic	Sample size	Mean Flow Rate $\bar{x}$	Population Standard Deviation σ
Pentobarbital	191	0.99	0.235
Urethane	52	1.47	0.314
Ketamine	16	0.99	0.164

a. Test the hypothesis that the true mean flow rate under the effects of Pentobarbital is greater than 0.96.

1. μ is the mean flow rate under the effects of Pentobarbital
2. $H_0: \quad \mu = 0.96 \qquad H_a: \quad \mu > 0.96$
3. The sample is a simple random sample from the population & the sample size is large enough to assume $\bar{x}$ is approximately normal
4. $Z = \frac{\bar{x} - 0.96}{\sigma/\sqrt{n}} = \frac{0.99 - 0.96}{0.235/\sqrt{191}} = 1.76$
5. $\text{p-value} = P(Z > 1.76) = 1 - 0.9608 = 0.0392$
6. Reject H_0 since $\text{p-value} < 0.05$
7. Conclude the mean flow rate under the effects of Pentobarbital is greater than 0.96

b. Test the hypothesis that the true mean flow rate under the effects of Urethane is less than 1.56.

1. μ is the mean flow rate under the effects of Urethane
2. $H_0: \quad \mu = 1.56 \qquad H_a: \quad \mu < 1.56$
3. The sample is a simple random sample from the population & the sample size is large enough to assume $\bar{x}$ is approximately normal
4. $Z = \frac{\bar{x} - 1.56}{\sigma/\sqrt{n}} = \frac{1.47 - 1.56}{0.314/\sqrt{52}} = -2.07$
5. $\text{p-value} = P(Z < -2.07) = 0.0192$
6. Reject H_0 since $\text{p-value} < 0.05$
7. Conclude the mean flow rate under the effects of Urethane is less than 1.56

c. Test the hypothesis that the true mean flow rate under the effects of Ketamine differs from 1.08.

1. μ is the mean flow rate under the effects of Ketamine
2. $H_0: \quad \mu = 1.08 \qquad H_a: \quad \mu \neq 1.08$
3. The sample is a simple random sample from the population & the sample size is not large enough; we must assume the Ketamine values are normal
4. $Z = \dfrac{\bar{x} - 1.08}{\sigma/\sqrt{n}} = \dfrac{0.99 - 1.08}{0.164/\sqrt{16}} = -2.20$
5. $\text{p-value} = 2 \times P(Z < -2.20) = 2 \times 0.0139 = 0.0278$
6. Reject H_0 since $\text{p-value} < 0.05$
7. Conclude the mean flow rate under the effects of Ketamine is not 1.08

2. A patient is said to be hypokalemic (low potassium in the blood) if the measured level of potassium is *3.5* or less. An individual's potassium level is not a constant, however, but varies from day to day. In addition, the measurement procedure itself has some variation. Suppose that the overall variation follows a normal distribution. Judy has a mean potassium level of *3.8* with a standard deviation of *0.2*. If she is measured on many days, on what proportion of days will the measurement suggest that Judy is hypokalemic?

$$P(X < 3.5) = P\left(Z < \frac{3.5 - 3.8}{0.2}\right) = P(Z < -1.5) = 0.0668$$

3. The heights (in inches) of adult males in the U.S. is normally distributed with mean μ. The average height of a random sample of *25* American adult males is found to be $\bar{x} = 69.72$ inches. It is known from previous research that the standard deviation of height is $\sigma = 4.0$. Find *90%* confidence interval for μ.

$$69.72 \pm 1.645 \, 4.00/\sqrt{25} \Rightarrow [68.404, 71.036]$$

4. An agronomist examines the cellulose content of a variety of alfalfa hay. Suppose that the cellulose content in the population has standard deviation of *8* mg/g. A sample of *15* cuttings has mean cellulose content *145* mg/g. A previous study claimed that the mean cellulose content was 140 mg/g, but the agronomist believes that the mean is higher than that figure. State H_0 and H_a, calculate the test statistic, compute the p-value, and state your conclusions using a significance level of 5%.

1. μ is the mean cellulose content of alfalfa hay
2. $H_0: \quad \mu = 140 \qquad H_a: \quad \mu > 140$
3. The sample is a simple random sample from the population &
   the sample size is not large; we must assume that cellulose content is is normal
4. $Z = \dfrac{\bar{x} - 140}{\sigma/\sqrt{n}} = \dfrac{145 - 140}{8/\sqrt{15}} = 2.42$
5. $\text{p-value} = P(Z > 2.42) = 1 - 0.9922 = 0.0078$
6. Reject H_0 since $\text{p-value} < 0.05$
7. Conclude the mean cellulose content is less than 140

5. A bottling company uses a filling machine to fill plastic bottles with cola. The bottles are supposed to contain *300* milliliters (*ml*). The company wants to estimate the mean amount of cola in a bottle with an error of less than *3 ml* with *90%* confidence. How large a simple random sample should the company take if it is known that the standard deviation is σ = *3*ml?

$$n \geq \left(\frac{Z\sigma}{ME}\right)^2 = \left(\frac{1.645(3)}{3}\right)^2 = 2.7 \qquad \Longrightarrow \qquad n = 3$$

6. You wish to estimate the mean weight of machine components of a certain type and you require a 92% confidence that the sample mean will be in error by no more than 0.004 grams. Find the sample size required. A pilot study showed that the population standard deviation is 0.09 grams.

$$n \geq \left(\frac{Z\sigma}{ME}\right)^2 = \left(\frac{1.75(0.09)}{0.004}\right)^2 = 1550.39 \qquad \Longrightarrow \qquad n = 1551$$

7. Suppose a large labor union wishes to estimate the mean number of hours per month a union member is absent from work. The union decides to sample *45* of its members at random and monitor their working time for *1* month. At the end of the month, the total number of hours absent from work is recorded for each employee. The mean of the sample is $\bar{x} = 9.6$ hours. Assume the population standard deviation is $\sigma = 6.4$.

   a. Determine a *95%* confidence interval for μ, the mean number of hours absent per month.

   A 95% confidence interval for μ is $\bar{x} \pm 1.96\, \sigma/\sqrt{n}$

   $9.6 \pm 1.96\, 6.4/\sqrt{45}$

   9.6 ± 1.87

   $[7.73, 11.47]$

b. An administrator claims that the mean number of hours absent per month is greater than *9*. Test this hypothesis using $\alpha = 0.05$.

1. μ is the mean number of hours absent per month

2. $H_0: \ \mu = 9 \qquad H_a: \ \mu > 9$

3. The sample is a simple random sample from the population & the sample size is large enough that $\overline{x}$ is approximately normal

4. $$Z = \frac{\overline{x} - 9}{\sigma / \sqrt{n}} = \frac{9.6 - 9}{6.4 / \sqrt{45}} = 0.63$$

5. p-value $= P(Z > 0.63) = 1 - 0.7356 = 0.2644$

6. Fail to Reject H_0 since p-value > 0.05

7. Conclude the mean number of hours absent each month is 9 (not greater than 9).

8. The diameters of bolts produced by a certain machine are normally distributed with a mean of 0.30 inches and a standard deviation of 0.01 inches. What percentage of bolts will have a diameter greater than 0.32 inches?

$$P(X > 0.32) = P\left(Z > \frac{0.32 - 0.30}{0.01}\right) = P(Z > 2.00) = 1 - 0.9772 = 0.0228$$

9. Over the last month a large supermarket chain has received many consumer complaints about the quantity of chips in *9*-ounce bags of a particular brand of potato chips. Suspecting that the complaints are merely the result of potato chips settling to the bottom of the bags during shipping but wanting to be able to assure its customers they are getting their money's worth, the chain decides to examine the next shipment of chips received by their largest store. Thirty-five *9*-ounce bags are randomly selected from the shipment, their contents are weighed, and the average of the sample turns out to be 8.95. The company who manufactures chips says the standard deviation of the weights of a bag of chips is $\sigma = 0.13$ ounces. Test the most likely hypothesis using $\alpha = 0.05$. What is the p-value of your test?

1. μ is the mean ounces of potato chips in the bags

2. $H_0: \ \mu = 9 \qquad H_a: \ \mu < 9$

3. The sample is a simple random sample from the population & the sample size is large enough that $\overline{x}$ is approximately normal

4. $$Z = \frac{\overline{x} - 9}{\sigma / \sqrt{n}} = \frac{8.95 - 9}{0.13 / \sqrt{35}} = -2.28$$

5. p-value $= P(Z < -2.28) = 0.0113$

6. Reject H_0 since p-value < 0.05

7. Conclude the customers are correct the mean ounces of potato chips in the bags is less than 9.

10. Suppose the replacement times for washing machines are normally distributed with a mean of *10.6* years and a standard deviation of *2* years. Find the replacement time that separates the top *18*% from the bottom *82*%.

$$C = \mu + Z\sigma = 10.6 + 0.92 * 2 = 12.44$$

11. A bottling company uses a filling machine to fill plastic bottles with cola. The bottles are supposed to contain *300* milliliters (ml). A simple random sample of six bottles had a sample average of *298* ml. It is known that the population's standard deviation is *3* ml. Give a *90*% confidence interval for the mean contents of six bottles chosen randomly from the population.

A 90% confidence interval for μ is $\bar{x} \pm 1.645 \, \sigma/\sqrt{n}$

$$298 \pm 1.645 \, 3/\sqrt{6}$$

$$298 \pm 2$$

$$[296, 300]$$

12. A grape seller claims that he always gives *a little extra* in order to insure that his customers are satisfied. The weights of *100* pallets of grapes were recorded. The average and standard deviation were calculated to be *37.7* and *5.4* pounds, respectively. Is there significant evidence that the average weight of all his pallets is more than the *35* pounds that his customers paid for? State the appropriate hypothesis tests, the p-value, and your conclusion.

1. μ is the mean pounds of a pallet of grapes

2. $H_0: \mu = 35 \qquad H_a: \mu > 35$

3. The sample is a simple random sample from the population & the sample size is large enough that $\bar{x}$ is approximately normal

4. $Z = \dfrac{\bar{x} - 35}{\sigma/\sqrt{n}} = \dfrac{37.7 - 35}{5.4/\sqrt{100}} = 5.00$

5. p-value $= P(Z > 5.00) < 0.0005$

6. Reject H_0 since p-value < 0.05

7. Conclude the grape seller does give a little extra and that the mean pounds per pallet are more than 35.