Lab #4
Probability – Sample Spaces, Events, and Central Limit Theorem

1. Consider the experiment in which you roll two fair dice.

   a. How many outcomes are in the sample space? Determine all the outcomes in the sample space. Are they equally likely?
   36 outcomes in the sample space; they are equally likely
   S = { 11, 12, 13, 14, 15, 16, 21, 22, 23, 24, 25, 26,
   31, 32, 33, 34, 35, 36, 41, 42, 43, 44, 45, 46,
   51, 52 ,53, 54, 55, 56, 61, 62, 63, 64, 65, 66 }

   b. Define the random variable X to be the sum of the two dice. Determine P(x), the distribution of the random variable, X?

x	2	3	4	5	6	7
P(x)	1/36	2/36	3/36	4/36	5/36	6/36

x	8	9	10	11	12
P(x)	5/36	4/36	3/36	2/36	1/36

   c. Define the random variable y to be the larger of the two dice. Determine P(y), the distribution of the random variable, y?

y	1	2	3	4	5	6
P(y)	1/36	3/36	5/36	7/36	9/36	11/36

2. The distribution of B.S. degrees conferred by major at a large university are

Major	Frequency
English	2,073
Mathematics	2,164
Chemistry	318
Physics	856
Liberal Arts	1,358
Business	1,676
Engineering	868
	9,313

   a. What is the probability that a randomly selected degree is in Liberal Arts?
   P(Liberal Arts) = 1358 / 9313 = 0.1458

   b. What is the probability that a randomly selected degree is not in Mathematics?
   P(Math) = 2164 / 9313 = 0.2324
   P(not Math) = 1 – 0.2324 = 0.7676

   c. What is the probability that a randomly selected degree is in Mathematics or English?
   P(Math or English) = P(Math) + P(English) = 2164 / 9313 + 2073 / 9313 = 0.0.4550

   d. What is the probability that a randomly selected degree is in Physics, Business, or Engineering?
   P(Physics, Business, or Engineering) = (856 + 1676 + 868) / 9313 = 0.3651

3. If you choose a household at random and let the random variable X be the number of persons living in the household, then the probability distribution of X is:

x	1	2	3	4	5	6	7	**Sum**
P(x)	0.25	0.30	0.20	**0.11**	0.10	0.03	0.01	**1.00**

a. Determine P(X = 4).
All probabilities must sum to one, therefore 0.11

b. Determine $P(X \geq 5)$.
$P(X \geq 5) = P(5) + P(6) + P(7) = 0.10 + 0.03 + 0.04 = 0.17$

c. Determine $P(X < 3)$.
$P(X < 3) = P(1) + P(2) = 0.25 + 0.30 = 0.55$

d. Determine the probability that at most 5 people live in the household.
P(at most 5) = 1 – P(more than 5) = 1 – 0.03 – 0.01 = 0.96

e. Determine the probability that at least three are living in the household.
P(at least three) = 1 – P(1) – P(2) = 1 – 0.25 – 0.30 = 0.45

f. Determine the probability that more that 6 people are living in the household.
P(7) = 0.01

g. Suppose A is the event that there are more than 2 people in the household. Determine P(A does not occur).
A = more than 2 people in household → (not A) = 1 or 2 people live in household
P(not A) = 0.25 + 0.30 = 0.55

4. A package of M&M's yielded *22* blue, *35* pink, *12* yellow, and *28* green M&M's. If a single M&M is randomly selected,

a. what is the probability that the M&M was purple?
zero, I wasn't trying to be tricky I just thought purple was in the list.

b. what is the probability that the M&M was blue?
22 / 97 = 0.2268

c. what is the probability that the M&M was pink or green?
(35 + 28) / 97 = 0.6495

d. If two M&M's are randomly selected, without replacement, what is the probability that the first is pink and the second is yellow?
(35 / 97) (12 / 96) = 0.0451

5. A hat contains *40* marbles. Of them, *16* are red and *24* are green. If one marble is randomly selected out of this hat what is the probability that this marble is

a. Red? **16 / 40 = 0.40**
b. Green? **24 / 40 = 0.60**

6. Assume that the random variable x has a continuous probability distribution that is Normal with a mean of 50 and a standard deviation of 14. Find the probabilities:

a. $$\begin{aligned} P(x<38) &= P\left(\frac{x-50}{14} < \frac{38-50}{14}\right) \\ &= P(z<-0.86) \\ &= 0.1949 \end{aligned}$$

b. $P(x > 59) = P\left(\frac{x-50}{14} > \frac{59-50}{14}\right)$

$= P(z > 0.64)$

$= 1 - P(z < 0.64)$

$= 1 - 0.7389 = 0.2611$

c. $P(25 < x < 68) = P\left(\frac{25-50}{14} < z < \frac{68-50}{14}\right)$

$= P(-1.79 < z < 1.29)$

$= P(z < 1.29) - P(z < -1.79)$

$= 0.9015 - 0.0367 = 08648$

d. If P(x > a) = 0.10, what is the value of a?

$P(x > a) = 0.10 \Rightarrow P(x < a) = 0.90$

$P\left(z < \frac{a-50}{14}\right) = 0.90$

$\Rightarrow \frac{a-50}{14} = 1.28$

$\Rightarrow a = 50 + 1.28 \times 14 = 67.92$

7. According to Opinion Research Corporation, the length of time spent in the shower by men follows a normal probability distribution with a mean of *11.4* min and a standard deviation of *1.8* minutes.

a. $P(x < 10) = P\left(\frac{x-11.4}{1.8} < \frac{10-11.4}{1.8}\right)$

$= P(z < -0.78)$

$= 0.2184$

b. What is the probability that the shower lasts at least *15* minutes?

$P(x \geq 15) = P\left(z \geq \frac{15-11.4}{1.8}\right)$

$= P(z \geq 2.00)$

$= 1 - P(z < 2.00)$

$= 1 - 0.9772 = 0.0228$

c. What is the probability that the shower lasts between *10* and *15* minutes?

$P(10 \leq x \leq 15) = P\left(\frac{10-11.4}{1.8} \leq z \leq \frac{15-11.4}{1.8}\right)$

$= P(-0.78 \leq z \leq 2.00)$

$= 0.9772 - 0.2184 = 0.7588$

8. According to the College Board, the scores on the SAT Math exam have a normal distribution with a mean of *500* and a standard deviation of *100*.

a. $P(x < 450) = P\left(\frac{x-500}{100} < \frac{450-500}{100}\right)$

$$= P(z < -0.50)$$

$$= 0.3085$$

b. What is the percentile for a student who scores *600* on the exam?

$$P(x < 600) = P\left(\frac{x-500}{100} < \frac{600-500}{100}\right)$$

$$= P(z < 1.00)$$

$$= 0.8413$$

c. What is the probability that a student will score at least *650* on the exam?

$$P(x \geq 650) = P\left(\frac{x-500}{100} \geq \frac{650-500}{100}\right)$$

$$= P(z \geq 1.50)$$

$$= 1 - 0.9332$$

$$= 0.0668$$

9. Suppose that the data in a population follow a *N(200, 5)* distribution.

a. What is the mean μ of this distribution? What is the standard deviation of this distribution?

$\mu = 100$, $\sigma = 5$

b. What percent of this population is less than *190*?

$$P(x < 190) = P\left(z < \frac{190-200}{5}\right) = P(z < -2.00) = 0.0228$$

c. What is P(192 < X < 199)?

$$P(192 < x < 199) = P\left(\frac{192-200}{5} < z < \frac{199-200}{5}\right)$$

$$= P(-1.60 < z < -0.20)$$

$$= 0.4207 - 0.0548$$

$$= 0.3659$$