Chapter 6

Paired Comparisons

The paired-sample test is one of the most sensitive in statistics and experiments are frequently designed to take advantage of this. Paired samples are those in which an observation in sample 1 is in some way closely related with an observation in sample 2. The most obvious way in which two observations can be related is if they are made on the same animal or plant. For example, the effect of alcohol on the metabolism of animals can best be tested by measuring the metabolism of the same animal both before ingesting alcohol (the "control") and then again after ingesting alcohol (the "treatment"). Obviously the two measurements are closely related since they involve the same animal (same genetic system).

In such cases where the data are paired they can be tabulated in pairs (before and after alcohol in our example), and the statistical test is conducted on the difference between the pairs. A mean population difference, μ_d, can be defined as $\mu_1 - \mu_2$. For a two-tailed test, the null hypothesis can be stated as Ho: $\mu_d = c$ and Ha: $\mu_d \neq c$ where c is the hypothetical population mean to which the paired sample is being compared. The hypothetical mean c can take any value but frequently is 0 since the null hypothesis is a statement of no difference or no effect of the treatment. Using μ_d instead of $\mu_1 - \mu_2$ emphasizes that the difference between two populations is also a normally distributed population.

The test statistic for the paired-sample is

$$t = \frac{(\bar{d} - c)}{s_{\bar{d}}} \qquad [6.1]$$

where $\bar{d}$ is the mean difference, c is the hypothetical value, and $s_{\bar{d}}$ is the standard error of the differences. To obtain these values the difference between **each** pair of observations is first calculated. These differences are then used to compute a mean difference, $\bar{d}$, and standard error of the differences ($s_{\bar{d}}$). The number of differences, which equals the number of pairs of data, is the sample size (*n*) and the degrees of freedom (*df*) are equal to n-1. The paired-sample t test is really a one-sample test, described in Chapter 5, based on the differences between a series of paired observations. Thus it is not really a new statistical procedure, but rather a new application of one already covered.

One-tailed hypotheses can be tested with paired samples using either Ho: $\mu_d \geq c$ and Ha: $\mu_d < c$, or Ho: $\mu_d \leq c$ and Ha: $\mu_d > c$. As always, you must decide whether a one-tailed test is appropriate **before** conducting the analysis. Confidence intervals can also be calculated for μ_d as previously described in Chapter 5.

Example Problem

Table 6.1 An investigator wished to know if there is any difference in the length (cm) of forelegs and hindlegs of deer, and collected the following data.

deer	1	2	3	4	5	6	7	8	9	10
foreleg	138	136	147	139	143	141	143	145	136	146
hindleg	142	140	144	144	142	146	149	150	142	148

I. STATISTICAL STEPS

A. Statement of Ho and Ha

Ho: mean difference in length = 0

Ha: mean difference in length ≠ 0

B. Statistical test

Paired-sample t test

C. Computation of descriptive and test statistics

The template was used to compute the descriptive statistics needed, and is shown to the right. The statistics needed include the mean, SE, 95% confidence limits, and range values for the foreleg, hindleg and the difference between the two limbs.

	A	B	C	D
1	title here	foreleg	hindleg	difference
2	n	10	10	10
3	mean	141.40	144.70	-3.30
4	SE	1.28	1.08	0.97
5	low	136.00	140.00	-6.00
6	high	147.00	150.00	3.00
7	Lower 95 CL	138.51	142.27	-5.49
8	Upper 95 CL	144.29	147.13	-1.11
9	One-sample t			
10	enter hypo mea	144.7	141.4	0
11	t	2.587	-3.068	3.414
12	P	0.029	0.013	0.008
13		foreleg	hindleg	difference
14	data here	138	142	-4
15		136	140	-4
16		147	144	3
17		139	144	-5
18		143	142	1
19		141	146	-5
20		143	149	-6
21		145	150	-5
22		136	142	-6
23		146	148	-2
24				

D. Determination of P for the test statistic

P(t test$_{[\alpha(2)=0.05,14]}$ = 3.41) = 0.008 .

Note also that the 95% confidence limits around the difference do not include zero. This clearly indicates that the mean difference is different than zero; in this case, the sample mean difference is significantly lower than zero.

E. Statistical Inference

Reject Ho and accept Ha since the P of the test statistic, the t value, is < 0.05

II. Biological Inference

In deer the hindleg is significantly longer than the foreleg by 2.3%. {Calculated as the percent difference between the two means: = (144.7-141.4)/144.7}

These data would be presented in a table and figure similar to those below.

Table 6.2. Mean difference between the length (cm) of fore and hind legs of 10 deer. Data include the 95% confidence limit (CL) and the t and P values from paired comparison test.

Sample	mean ± SE	95% CL Low High	Range Low High	t	P
foreleg	141.4 ± 1.3	138.5 - 144.3	136 -147		
hindleg	144.7 ± 1.1	142.3 - 147.1	140 - 150		
difference	-3.3 ± 1.0	-5.5 - -1.1	-6 - 3	3.414	0.008*

*significant difference

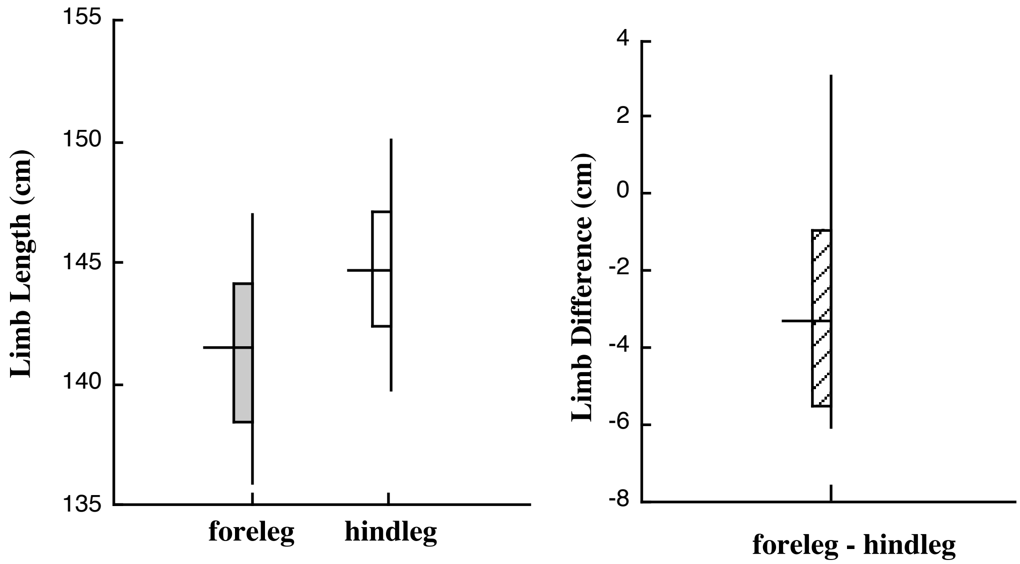

Figure 6.1. Foreleg and hindleg lengths of 10 deer, and the difference between the two. Horizontal lines indicate the means, vertical lines indicate the ranges, and the boxes span the upper and lower 95% confidence limits.

Problem Set – Paired Comparisons

You should be able to analyze paired comparison datasets using both hand calculations & the excel template. You should also recognize when a paired comparison ttest is appropriate to use. You will not turn these in.

6.1) A new fertilizer was developed to increase crop yields by 5 bu·acre^{-1} over that of an old fertilizer. Crops were grown in 9 pairs of test plots with each pair of plots selected for identical environmental conditions. Analyze and interpret the data (units, bu·acre^{-1}).

plot	1	2	3	4	5	6	7	8	9
old fertilizer	60.6	66.6	64.9	61.8	61.7	67.2	62.4	61.3	56.7
new fertilizer	67.4	72.8	68.4	66.0	70.8	69.6	67.2	68.9	62.6

6.2) For several days concentrations (nanogram·cubic meter^{-1}) of soil particles and of combustion byproducts (hydrocarbons) were determined in air sampled at the same site in Bakersfield to see if they differed in concentration. Analyze and interpret.

day	1	2	3	4	5	6	7	8	9	10	11
soil particles	104	116	84	77	61	84	81	72	61	97	84
combustion	108	118	89	71	66	83	88	76	68	96	81

6.3) Oxygen consumption (microliters·g hr^{-1}) was measured in twelve lizards before and after thyroxin injections. (Thyroxin is a hormone known to be responsible for increasing metabolism). Analyze and interpret.

Lizard	Before	After
1	26.3	28.0
2	24.2	25.7
3	23.1	22.9
4	21.6	22.4
5	24.5	25.2
6	23.3	25.4
7	21.8	23.0
8	22.9	23.4
9	25.7	25.5
10	24.1	24.8
11	25.8	25.6
12	26.1	25.7