

1. For each of the following pair of fractions, replace the semicolon with the correct symbol $<$, $=$ or $>$, to make a true statement. If you don't show the reason why, your answer will be marked wrong even if correct.

(a) $\frac{7}{8}; \frac{5}{6}$

Solution. Because $7 * 6 = 42 > 40 = 8 * 5$, we have $\frac{7}{8} > \frac{5}{6}$.

(b) $2\frac{4}{5}; 2\frac{3}{6}$

Solution. Because $4 * 6 = 24 > 15 = 3 * 5$, we have $\frac{4}{5} > \frac{3}{6}$. Hence, $2\frac{4}{5} > 2\frac{3}{6}$.

(c) $-\frac{7}{8}; -\frac{4}{5}$

Solution. Rewrite $-\frac{7}{8} = \frac{-7}{8}$ and $-\frac{4}{5} = \frac{-4}{5}$. Because $(-7) * 5 = -35 < -32 = (-4) * 8$, we have $-\frac{7}{8} < -\frac{4}{5}$.

2. Martine bought $8\frac{3}{4}$ yd of fabric. She wants to make a skirt using $1\frac{7}{8}$ yd, pants using $2\frac{3}{8}$ yd, and a vest using $1\frac{2}{3}$ yd. How much fabric will be left over?

Solution. To do the garments, Martine will use

$$\begin{aligned}
 1\frac{7}{8} + 2\frac{3}{8} + 1\frac{2}{3} &= \left(1 + \frac{7}{8}\right) + \left(2 + \frac{3}{8}\right) + \left(1 + \frac{2}{3}\right) \\
 &= \left(\frac{1}{1} + \frac{7}{8}\right) + \left(\frac{2}{1} + \frac{3}{8}\right) + \left(\frac{1}{1} + \frac{2}{3}\right) \\
 &= \left(\frac{1 * 8 + 7 * 1}{1 * 8}\right) + \left(\frac{2 * 8 + 1 * 3}{1 * 8}\right) + \left(\frac{1 * 3 + 2 * 1}{1 * 3}\right) \\
 &= \left(\frac{8 + 7}{8}\right) + \left(\frac{16 + 3}{8}\right) + \left(\frac{3 + 2}{3}\right) \\
 &= \left(\frac{15}{8}\right) + \left(\frac{19}{8}\right) + \left(\frac{5}{3}\right) \\
 &= \left(\frac{15 + 19}{8}\right) + \left(\frac{5}{3}\right) = \left(\frac{34}{8}\right) + \left(\frac{5}{3}\right) \\
 &= \left(\frac{17}{4}\right) + \left(\frac{5}{3}\right) = \left(\frac{17 * 3 + 5 * 4}{3 * 4}\right) \\
 &= \left(\frac{51 + 20}{12}\right) = \frac{71}{12}
 \end{aligned}$$

Thus, the fabric to be left over will be:

$$\begin{aligned}
 8\frac{3}{4} - \frac{71}{12} &= \left(8 + \frac{3}{4}\right) - \frac{71}{12} \\
 &= \left(\frac{8}{1} + \frac{3}{4}\right) - \frac{71}{12} \\
 &= \left(\frac{8 * 4 + 1 * 3}{1 * 4}\right) - \frac{71}{12} \\
 &= \left(\frac{32 + 3}{4}\right) - \frac{71}{12} \\
 &= \frac{35}{4} - \frac{71}{12} \\
 &= \frac{35 * 3}{4 * 3} - \frac{71}{12} \\
 &= \frac{105}{12} - \frac{71}{12} \\
 &= \frac{105 - 71}{12} \\
 &= \frac{34}{12} = \frac{17}{6} = 2\frac{5}{6},
 \end{aligned}$$

or $2\frac{5}{6}$ yd left over.

3. For each of the following, find two rational numbers between the given fractions.

(a) $\frac{3}{4}; \frac{4}{7}$.

Solution. For the first rational in between, try $\frac{3+4}{4+7} = \frac{7}{11}$. For the second one,

we can try either $\frac{3+7}{4+11} = \frac{10}{15}$, or $\frac{7+4}{11+7} = \frac{11}{18}$.

(b) $-\frac{7}{8}; -\frac{4}{5}$.

Solution. Rewrite $-\frac{7}{8} = \frac{-7}{8}$ and $-\frac{4}{5} = \frac{-4}{5}$. For the first rational in between,

try $\frac{(-7) + (-4)}{8 + 5} = \frac{-11}{13}$. For the second one, we can try either $\frac{(-7) + (-11)}{8 + 13} =$

$\frac{-18}{21}$, or $\frac{(-11) + (-4)}{13 + 5} = \frac{-15}{18}$.

4. A person bought $29\frac{1}{2}$ yd of fabric to make doll uniforms. Each uniform requires $\frac{3}{4}$ yd of fabric. How many uniforms can be made? How much material will be left over?

Solution. If each uniform requires $\frac{3}{4}$ yd of fabric, the question is how many times $\frac{3}{4}$

fits into $29\frac{1}{2} = 29 + \frac{1}{2} = \frac{29 * 2}{2} + \frac{1}{2} = \frac{59}{2}$? The answer is then

$$\begin{aligned}\frac{59}{2} \div \frac{3}{4} &= \frac{59 * 4}{2 * 3} = \frac{236}{6} = 39 + \frac{2}{6} \\ &= 39 + \frac{1}{3} = 39\frac{1}{3}.\end{aligned}$$

Thus, 39 uniforms can be made, $\frac{1}{3} \times \frac{3}{4} = \frac{3}{12} = \frac{1}{4}$ yd of fabric left over.

5. Jasmine is reading a book. She has finished $\frac{3}{4}$ of the book and has 82 pages left to read. How many pages has she read?

Solution. Let ℓ be the total number of pages in the book. Then, Jasmine has read $\frac{3}{4}\ell$ pages. Since she still has $\frac{1}{4}\ell$ pages to read and these total 82, we have

$$\frac{1}{4}\ell = 82 \iff \frac{\ell}{4} = \frac{82}{1}$$

which implies

$$\ell = \frac{82 * 4}{1} = \frac{328}{1} = 328.$$

Thus, Jasmine has read $\frac{3}{4}\ell = \frac{3}{4}328 = \left(\frac{3}{4}\right)\left(\frac{328}{1}\right) = \left(\frac{3 * 328}{4 * 1}\right) = \frac{984}{4} = 246$ pages.