CALIFORNIA STATE UNIVERSTIY, BAKERSFIELD
MATHEMATICS FIELD DAY 2006
Team Medley, Varsity Level

Each correct answer is worth five points. Partial credit may be given. An unanswered question is given zero points.

No calculators are allowed. You have 50 minutes to complete the Exam. When the exam is over, give only one set of answers per team to the procter.

Elegance of solutions may affect score and may be used to break ties.

(1) A sequence of n similar triangles are such that the sum of the sides of any triangle after the first is half of the sum of the sides of the preceeding one. Find the ratio of the area of the first triangle to the n-th one, for any positive integer n, express your answer in the simplest fraction.

(2) Let ab be a non-zero number in base 7. The number ba, if considered as a number in base 8, is three times as large. Find the original number in base 10.

(3) Find the minimum value of

$$f(x) = \sqrt{2x^2 + \sqrt{2}x + \frac{1}{8}}$$

(4) How many integer solutions (that is, x and y are both integers) are there to the equation

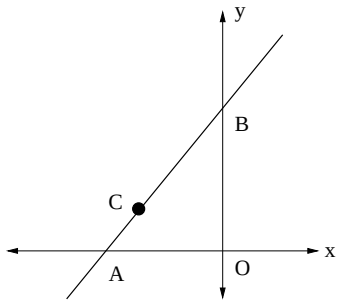
$$x^2 + 9y^2 + 4 = 4x + 18y \ ?$$

(5) A sequence of numbers $x_1, x_2, x_3, \ldots$ is defined by $x_1 = 1$ and

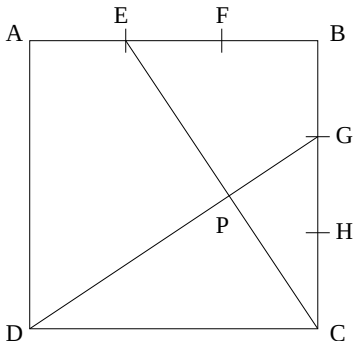
$$x_{n+1} = \frac{n}{n+1}\, x_n + 1, \quad \text{for all } n \geq 1$$

Find, and **justify**, a formula for x_n, where n is any positive integer.

(6) In the following diagram the line AB passes through the point $C(-2, 1)$. Suppose the area of ΔOAB is 4. Find the slope of the line AB.



(7) In the following diagram of a unit square, E, F trisect AB and G, H trisect BC, what is the area of ΔGPC?



(8) Grandpa's current age is 7 times the age of Mary. A few years later, his age will be exactly 6 times the age of Mary. A few year more later, his age will be exactly 5 times the age of Mary. Then, a few years after that, his age will be exactly 4 times that of Mary. A few more years after that, his age will be exactly 3 times that of Mary. Eventually, his age will be exactly twice that of Mary. Assume grandpa is under 100 years old, what is his current age?