CALIFORNIA STATE UNIVERSTIY, BAKERSFIELD
MATHEMATICS FIELD DAY 2006
Team Medley, Junior Varsity Level

Each correct answer is worth five points. Partial credit may be given. An unanswered question is given zero points.

No calculators are allowed. You have 50 minutes to complete the Exam. When the exam is over, give only one set of answers per team to the procter.

Elegance of solutions may affect score and may be used to break ties.

(1) A sequence of n similar triangles are such that the sum of the sides of any triangle after the first is half of the sum of the sides of the preceeding one. Find the ratio of the area of the first triangle to the n-th one, for any positive integer n, express your answer in the simplest fraction.

(2) Let ab be a non-zero number in base 7. The number ba, if considered as a number in base 8, is three times as large. Find the original number in base 10.

(3) Blood samples of 100 people were tested. The A, B and Rh antigens were found in the blood of 34, 35 and 75 people, respectively. None of the antigens were found in 13 samples. Eleven samples contained the A and Rh antigens only, 8 contained the B and Rh antigens only, 6 contained the B antigen only and 4 contained the A and B antigens only. How many samples contain exactly one antigen?

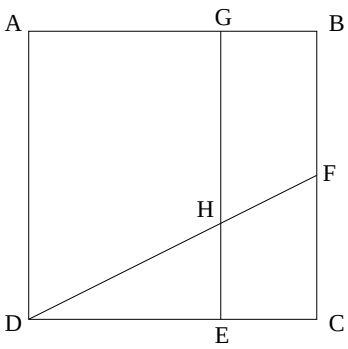
(4) Find all the solutions to the following system of equations:

$$\begin{aligned} x^2 + y^2 &= 4 \\ 5x^2 - y^2 &= 2 \end{aligned}$$

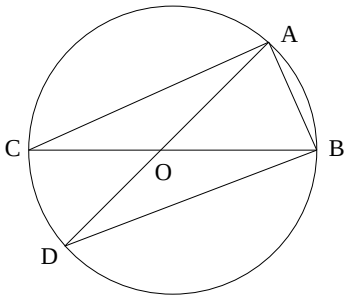
(5) How many integer solutions (that is, x and y are both integers) are there to the equation

$$x^2 + y^2 + 4 = 2x + 4y \ ?$$

(6) In the following unit square $ABCD$, if $ED = 2EC$ and F bisect BC, find the area of $BFHG$.



(7) In the following diagram, if O is the center of the circle, and $\angle ADB = 30°$. Show that ΔOAB is an equilateral triangle.



(8) Grandpa's current age is 7 times the age of Mary. A few years later, his age will be exactly 6 times the age of Mary. A few year more later, his age will be exactly 5 times the age of Mary. Then, a few years after that, his age will be exactly 4 times that of Mary. A few more years after that, his age will be exactly 3 times that of Mary. Eventually, his age will be exactly twice that of Mary. Assume grandpa is under 100 years old, and all ages are integers, what is his current age?