

**CALIFORNIA STATE UNIVERSITY, BAKERSFIELD
MATHEMATICS FIELD DAY 2006
Individual Medley, Varsity Level**

There are 25 problems. You will have 50 minutes. You will only have to turn in the answer sheet – you may keep the test. Each correct answer is worth one point. Each incorrect answer will receive a one-fourth point penalty. If you believe there is an error of some sort in a particular problem, please quietly discuss it with one of the proctors.

For each of the questions, blacken the appropriate circle on the answer sheet. When the exam is over, give your answer sheet to the proctor.

Calculators of any kind are not allowed.

GOOD LUCK!

-
- (1) The solution to $|2x - 3| \leq 7$ is
- (A) $x \geq -2$
 - (B) $x \geq 5$
 - (C) $-2 \leq x \leq 5$
 - (D) $x \leq -2$ or $x \geq 5$
 - (E) $-5 \leq x \leq 2$
- (2) An urn contains 4 blue balls, 5 red balls, 6 green balls. Two balls are drawn in succession, without replacing the first ball. What is the probability that there is one red ball, and one green ball?
- (A) $1/7$
 - (B) $2/15$
 - (C) $2/7$
 - (D) $4/15$
 - (E) $7/15$
- (3) The total number of integers between 1 and 101 that are multiples of 4 or 6, or both is
- (A) 37
 - (B) 41
 - (C) 33
 - (D) 29
 - (E) 25
- (4) The largest possible value for the function $f(x) = -2x^2 + 4x + 8$ is
- (A) 10
 - (B) 8
 - (C) 2
 - (D) 5
 - (E) 0

- (5) Solve the system of equations

$$\frac{2}{x} + \frac{3}{y} = 2, \quad \frac{4}{x} - \frac{9}{y} = -1$$

- (A) $(2, -3)$
(B) $(1/2, -3/2)$
(C) $(1, 3)$
(D) $(-1, 3/4)$
(E) $(2, 3)$
- (6) If $x < 2$, then $|2 + |x - 5||$ is
- (A) $x - 3$
(B) $3 - x$
(C) $7 - x$
(D) $x - 7$
(E) $-3 - x$
- (7) A point P is randomly selected from a triangular region bounded by $(0, 0)$, $(0, 4)$ and $(4, 0)$. What is the probability that P is at most one unit away from at least one of the axes?
- (A) 1
(B) $3/4$
(C) $1/2$
(D) $1/4$
(E) 0
- (8) Let a and r be positive real numbers, for the following set of 10 numbers

$$a - r, a, a + r, a + 2r, \dots, a + 7r, a + 8r$$

What is the inter-quartile range?

- (A) $5r$
(B) $a + 4.5r$
(C) r
(D) a
(E) $4.5r$

(9) The area enclosed by the graphs $y = |x|$ and $y = \sqrt{1 - x^2}$ is

- (A) $1/2$
- (B) $\pi/2$
- (C) π
- (D) 1
- (E) $\pi/4$

(10) How many distinct real roots are there in the equation

$$x^4 - 3x^3 - 6x^2 = 0?$$

- (A) 0
- (B) 1
- (C) 2
- (D) 3
- (E) 4

(11) The domain of the function $f(x) = \frac{e^{\ln x}}{\sqrt{x-1}}$ is

- (A) $(0, \infty)$
- (B) $(-\infty, \infty)$
- (C) $(1, \infty)$
- (D) $[0, \infty)$
- (E) $[1, \infty)$

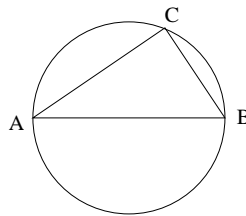
(12) One value of $\cos(\sin^{-1}(5/13)) =$

- (A) $12/13$
- (B) $5/12$
- (C) $5/13$
- (D) $-12/13$
- (E) $-5/12$

- (13) Suppose $F(n)$ is a function such that $F(5) = F(4) = 2$, $F(3) = 1$ and

$$F(n) = \frac{4F(n+1) + 2F(n+2)}{F(n+3)}, \text{ then } F(1) =$$

- (A) 2
(B) 7
(C) 3
(D) 9
(E) Cannot be determined
- (14) In the following diagram, suppose AB is the diameter of the circle with radius 1, then the maximum possible area for the triangle $\triangle ABC$ is



- (A) 1
(B) $1/2$
(C) π
(D) $\pi/2$
(E) $\sqrt{2}$
- (15) Let $i^2 = -1$, and $z = 1 + \sqrt{3}i$, the angle between z and \bar{z} on the complex plane is
- (A) 0
(B) $\pi/2$
(C) $2\pi/3$
(D) $3\pi/4$
(E) 2π

(16) A coin is tossed 7 times. The total number of ways to obtain a total of 3 heads is

- (A) 7
- (B) 8
- (C) 14
- (D) 21
- (E) 35

(17) The unit digit in $7^{2^{12}}$ is

- (A) 1
- (B) 3
- (C) 5
- (D) 7
- (E) 9

(18) The number $(4c3)$ in base 5 is divisible by 6. A possible value for c is

- (A) 0
- (B) 1
- (C) 2
- (D) 3
- (E) 4

(19) Let \mathbb{R} be the set of real numbers, \mathbb{Q} be the set of rational numbers, \mathbb{Z} be the set of integers, then $(\mathbb{R} - \mathbb{Q}) \cap \mathbb{Z}$ is

- (A) \mathbb{R}
- (B) \mathbb{Q}
- (C) \mathbb{Z}
- (D) \emptyset
- (E) none of the above

(20) What is the center of the circle

$$x^2 + y^2 - 4x + 6y - 3 = 0?$$

- (A) $(2, -3)$
- (B) $(-2, 3)$
- (C) $(-4, 6)$
- (D) $(2, 3)$
- (E) $(2, -2)$

(21) Two cars A and B depart an intersection. A is moving due south at 20 mph, B is moving due northeast at $60\sqrt{2}$ mph. What is the speed of B relative to A ?

- (A) $80\sqrt{2}$ mph
- (B) 80 mph
- (C) $40\sqrt{2}$ mph
- (D) 100 mph
- (E) Cannot be determined

(22) If

$$3f(t) = \frac{12t^2 - 6f(t)}{4t}, \text{ then } f(1) =$$

- (A) 4
- (B) $1/3$
- (C) $5/3$
- (D) 2
- (E) $2/3$

(23) The range of the function $f(x) = \text{Arctan } x + \frac{1}{2}\text{Arcsin } x$ is

- (A) $[-\pi, \pi]$
- (B) $[-3\pi/4, 3\pi/4]$
- (C) $[-\pi/2, \pi/2]$
- (D) $[-3\pi/2, 3\pi/2]$
- (E) $[-\pi/4, \pi/4]$

(24) A solution to the following equation is

$$6^{2/x} - 9 \cdot (6^{1/x}) + 18 = 0$$

- (A) $1 + \frac{\log_{10} 2}{\log_{10} 3}$
- (B) $1 - \log_{10} 6$
- (C) 6
- (D) $\log_{10} 3$
- (E) $-1 - \frac{\log_{10} 2}{\log_{10} 3}$

(25) The difference between the largest and the smallest possible value for the function $f(x) = \sin x + \cos x$ is

- (A) 1
- (B) 2
- (C) $\sqrt{2}$
- (D) $2\sqrt{2}$
- (E) $2\sqrt{3}$