

**CALIFORNIA STATE UNIVERSITY, BAKERSFIELD
MATHEMATICS FIELD DAY 2007**

Team Medley, Varsity Level

Each correct answer is worth five points. Partial credit may be given. An unanswered question is given zero points.

No calculators are allowed. You have 50 minutes to complete the Exam. When the exam is over, give only one set of answers per team to the proctor.

Elegance of solutions may affect score and may be used to break ties.

1. If $(x - 1)^2$ is a factor of $ax^4 + bx^3 + 1$, determine the values of a and b .

2. Find the minimum value of the function

$$f(x) = \sqrt{3x^2 - \sqrt{3}x + 1}$$

3. Let a , b , and c be real numbers. If two of the roots of the equation $x^4 + ax^2 + bx + c = 0$ are 3 and $1 - i$ (where $i^2 = -1$), find the value of c .

4. Consider a square whose side is of length of 2 inches. If there are 5 points inside the square, prove that there exist at least one pair of the points such that the distance between the two points is less than $\sqrt{2}$.

5. A point P is randomly selected from a triangular region whose vertices are $(0, 0)$, $(4a, 0)$, and $(4a, 3a)$, where a is a positive real number. What is the probability that P is at least a units away from each of vertices of the triangle?

6. Evaluate the following sum

$$\sum_{k=1}^n \frac{1}{k!(k+2)} = \frac{1}{1!3} + \frac{1}{2!4} + \frac{1}{3!5} + \cdots + \frac{1}{n!(n+2)}$$

7. There are 8 distinct balls labeled with letters a through h in an urn. Four balls are drawn from the urn and arranged in a row. If the ball a is selected and the ball b is *not*, how many different arrangements are possible?

8. Let C_k denote the circle whose equation is $x^2 + y^2 + 2kx - 2ky - 2k - 4 = 0$, where k is a real number. Are there any points through which the circle C_k passes for every k ? If so, find them. If not prove it.