1. The average (arithmetic mean) age of 28 members in a club was 25. After two new members whose age difference was 6, joined the club, the average age of the members of the club didn’t change. What was the age of the younger of the new members?

\[
\frac{x_1 + x_2 + \cdots + x_{30}}{28} = 25 \implies x_1 + \cdots + x_{28} = 700
\]

\[
x_{29} = a, \quad x_{30} = a + 6
\]

\[
\frac{x_1 + x_2 + \cdots + x_{30}}{30} = 25 \implies x_1 + \cdots + x_{28} + 2a + 6 = 750
\]

\[
\therefore 2a + 6 = 50 \implies 2a = 44
\]

\[
\therefore a = 22
\]

2. Find the minimum value of the function

\[
f(x) = \sqrt{3x^2 - \sqrt{3}x + 1}
\]

vertex at \(-\frac{b}{2a} = \frac{\sqrt{3}}{6}\)

So, min is

\[
\sqrt{3 \cdot \frac{\sqrt{3}}{6} - \sqrt{3} \cdot \frac{\sqrt{3}}{6} + 1}
\]

\[
= \sqrt{\frac{1}{4} - \frac{1}{2} + 1}
\]

\[
= \sqrt{\frac{3}{4}}
\]

\[
= \frac{\sqrt{3}}{2}
\]

\[
3x^2 - \sqrt{3}x + 1 = 3 \left[ x^2 - \frac{\sqrt{3}}{3}x + \left( \frac{\sqrt{3}}{6} \right)^2 - \left( \frac{\sqrt{3}}{6} \right)^2 \right] + 1
\]

\[
= 3 \left[ (x - \frac{\sqrt{3}}{6})^2 - \frac{3}{36} \right] + 1
\]

\[
= 3 \left( x - \frac{\sqrt{3}}{6} \right)^2 - \frac{1}{4} + 1
\]

\[
= 3 \left( x - \frac{\sqrt{3}}{6} \right)^2 + \frac{3}{4}
\]

\[
\therefore \text{min is } \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}
\]
3. If \( x + \frac{1}{x} = 1 \), find the value of \( x^3 + \frac{1}{x^3} \).

\[
x + \frac{1}{x} = 1
\]

\[
1 = \left( x + \frac{1}{x} \right)^3 = x^3 + 3x^2 \cdot \frac{1}{x} + 3x \cdot \frac{1}{x^2} + \frac{1}{x^3}
\]

\[
= x^3 + 3x + 3 \cdot \frac{1}{x} + \frac{1}{x^3}
\]

\[
= (x^3 + \frac{1}{x^3}) + 3 \left( x + \frac{1}{x} \right)
\]

\[
= (x^3 + \frac{1}{x^3}) + 3 \cdot 1
\]

\[
\therefore \quad x^3 + \frac{1}{x^3} = 1 - 3 = -2
\]

4. Let \( ab_8 \) and \( ba_6 \) denote two-digit integer representations in base 8 and 6 respectively, where \( a \) and \( b \) are positive integers less than 6. If 5 is a common factor of \( ba_6 \) and \( ab_8 - ba_6 \), what is the value of \( ab_8 \) in base 10?

\[
\text{1. } 5 \mid (6b+a) \quad \text{2. } 5 \mid \{ (8a+b) - (6b+a) \}
\]

\[
\text{5 \mid (7a - 5b). Also, } 5 \mid 5b.
\]

\[
\therefore \quad 5 \mid 7a \quad \Rightarrow \quad 5 \mid a. \quad \text{Since } 1 \leq a \leq 5,
\]

\[
\therefore \quad a = 5
\]

\[
\text{1. } 5 \mid (6b+a) \quad \Rightarrow \quad 5 \mid 6b \quad \Rightarrow \quad 5 \mid b
\]

\[
\therefore \quad b = 5
\]

\[
\therefore \quad ab_8 = 8a+b = 8 \cdot 5 + 5 = 45
\]
5. Find the value $x$ in the following figure.

Note that $\angle A = 90^\circ$

$\triangle ABC \cong \triangle DEC$

\[ \frac{9}{x} = \frac{15}{10} \]

\[ 9 \cdot 10 = 15 \cdot x \]

\[ x = 6 \]

6. Consider a square whose side is of length of 2 inches. If there are 5 points inside the square, prove that there exist at least one pair of the points such that the distance between the two points is less than $\sqrt{2}$ inches.

If you subdivide the square into 4 equal squares, then at least one of them has at least two points. The distance between these points is less than $\sqrt{2}$ (which is the length of the diagonal of the unit square).
7. Find the value of \( x \) in the following figure.

\[
x^2 + a^2 = 10^2
\]
\[
x^2 + (21-a)^2 = 17^2
\]
\[
\therefore \quad x^2 + a^2 - 42a + 441 = 289
\]
\[
\Rightarrow \quad 42a = 252 \quad \Rightarrow \quad a = 6
\]
\[
\therefore \quad x^2 = 10^2 - 6^2 = 64
\]
\[
\therefore \quad x = 8
\]

8. There are 8 distinct balls labeled with letters \( a \) through \( h \) in an urn. Four balls are drawn from the urn and arranged in a row. If the ball \( a \) is selected and the ball \( b \) is not, how many different arrangements are possible?

- (\( \circ \) \( \circ \) \( \circ \) \( \circ \)) \( \circ \) \( \circ \) \( b \)
- \# of ways of choosing the other 3 balls from the remaining 6 balls (after \( a \) is selected & \( b \) is discarded)
  \[
  = 6 \text{C}_3 = \frac{6! \cdot 5!}{3! \cdot 3!} = 20
  \]
- \# of arrangements of 4 distinct balls
  \[
  = 4! = 4 \cdot 3 \cdot 2 \cdot 1 = 24
  \]
- Total \# of arrangements = \( (20)(24) = 480 \)