

**CALIFORNIA STATE UNIVERSITY, BAKERSFIELD
MATHEMATICS FIELD DAY 2007**

Individual Medley, Varsity Level

There are 25 problems. You will have 50 minutes. You will only have to turn in the answer sheet – you may keep the test. Each correct answer is worth one point. Each incorrect answer will receive a one-fourth point penalty. If you believe there is an error of some sort in a particular problem, please quietly discuss it with one of the proctors.

For each of the questions, fill in the appropriate circle on the answer sheet. When the exam is over, give your answer sheet to the proctor.

Calculators of any kind are not allowed.

GOOD LUCK!

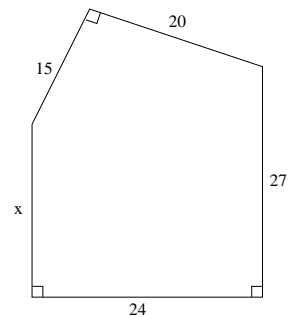
-
1. If $-1 < x < 3$, then $|x - 3| + |x + 1|$ equals
- (A) $2x - 2$
 - (B) 4
 - (C) $4 - x$
 - (D) $2 - 2x$
 - (E) None of the above
2. The domain of the function $f(x) = \frac{\sqrt{x-2}}{e^{\ln(5-x)}}$ is
- (A) $(2, 5)$
 - (B) $(2, \infty)$
 - (C) $[2, 5) \cup (5, \infty)$
 - (D) $[2, 5)$
 - (E) None of the above
3. Suppose $f(n)$ is a function such that $f(1) = 1$, $f(2) = 2$, and $f(n+2) = f(n) + 2f(n+1)$ for all natural numbers n . Then $f(5)$ equals
- (A) 5
 - (B) 12
 - (C) 15
 - (D) 23
 - (E) None of the above
4. The unit digit of 1357^{39} is
- (A) 3
 - (B) 7
 - (C) 9
 - (D) 1
 - (E) None of the above

5. If $\frac{x-3y}{x+y} = 7$, what is the value of $\frac{y}{x}$?
- (A) 1
(B) $\frac{3}{4}$
(C) $-\frac{2}{3}$
(D) $-\frac{3}{5}$
(E) It cannot be determined.
6. The value of $\cos^{-1}\left(\cos \frac{7\pi}{5}\right)$ is
- (A) $\frac{7\pi}{5}$
(B) $\frac{3\pi}{5}$
(C) $-\frac{7\pi}{5}$
(D) $\frac{2\pi}{5}$
(E) None of the above
7. The area of the region enclosed by the graph of $|x| + |y| = 1$ is
- (A) 1
(B) $\sqrt{2}$
(C) 2
(D) 3
(E) None of the above
8. Let $z = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i$, where $i^2 = -1$. What is z^{33} ?
- (A) $\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i$
(B) $-\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i$
(C) $-\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}i$
(D) i
(E) None of the above

9. If the points $(-2, 5)$ and $(5, 1)$ on the graph of $y = f(x)$, where f is a one-to-one function, $f^{-1}(5)$ equals
- (A) 1
 - (B) -2
 - (C) -1
 - (D) 5
 - (E) None of the above
10. Find $f(-2)$ where the function f is a cubic function whose graph has the x -intercepts -3 , -1 , and 2 and the y -intercept -24 .
- (A) 4
 - (B) -6
 - (C) -12
 - (D) 16
 - (E) None of the above
11. In how many different ways can 8 people be divided into two groups, one with 3 people and the other with 5 people?
- (A) 15
 - (B) 56
 - (C) 336
 - (D) 40320
 - (E) None of the above
12. If the function $f(x) = x^2 - 3$ is defined for $-2 \leq x \leq 2$, the maximum value of $|f(x)|$ is
- (A) 0
 - (B) 1
 - (C) 2
 - (D) 3
 - (E) None of the above

13. Let $f(x) = 2 \cos\left(1 - \frac{\pi x}{3}\right)$. What is the maximum possible value of $f(\alpha) - f(\beta)$?
- (A) 6
(B) 4
(C) 2
(D) 1
(E) None of the above
14. A point P is randomly selected from a triangular region bounded by $(0, 0)$, $(4, 0)$ and $(0, 4)$. What is the probability that P is at least one unit away from both of the axes?
- (A) $\frac{3}{4}$
(B) $\frac{1}{2}$
(C) $\frac{1}{4}$
(D) $\frac{1}{8}$
(E) None of the above
15. Let $f(x) = 2x^2 + 3x + 1$. For which value of k will the graph of $y = f(x - k)$ be symmetric with respect to the y -axis?
- (A) $\frac{3}{2}$
(B) $-\frac{3}{2}$
(C) $\frac{3}{4}$
(D) $-\frac{3}{4}$
(E) None of the above
16. The height of a cone is equal to the radius of its base. The radius of a sphere is equal to the radius of the base of the cone. The ratio of the volume of the cone to that of the sphere is
- (A) 1
(B) $\frac{1}{2}$
(C) $\frac{1}{3}$
(D) $\frac{1}{4}$
(E) None of the above

17. The exact value of $\tan[\sin^{-1}(\frac{12}{13})]$ is
- (A) $\frac{5}{12}$
(B) $\frac{12}{13}$
(C) $\frac{12}{5}$
(D) $\frac{12}{13}$
(E) None of the above
18. For points (x, y) on the circle $x^2 + y^2 = 1$, the maximum value of $x^2 + y$ is
- (A) 0
(B) 1
(C) 2
(D) $\frac{4+\sqrt{2}}{2}$
(E) None of the above
19. The average and standard deviation of the grades of a math exam were 67 and 15, respectively. If every grade is raised by 7, what is the standard deviation of the new grades?
- (A) 7
(B) 15
(C) 22
(D) 74
(E) None of the above
20. What is the area of the pentagon shown here with sides of length 15, 20, 27, 24, and x inches?



- (A) 798 in^2
(B) 714 in^2
(C) 688 in^2
(D) 648 in^2
(E) It cannot be determined without the value of x

21. Suppose that 320 grams of a radioactive material was put in a container 3 years ago. If 160 grams of the material was found to remain in the container today, when will 20 grams of the material remain in the container?.
- (A) 6 years later
(B) 9 years later
(C) 12 years later
(D) 24 years later
(E) None of the above
22. The minute hand and hour hand of a clock have lengths 3 and $\sqrt{2}$ inches, respectively. What is the distance between the tips of the hands at 4:30?
- (A) 2 in
(B) $\sqrt{11}$ in
(C) $\sqrt{7}$ in
(D) $\sqrt{5}$ in
(E) None of the above
23. Suppose that x and y are the real numbers such that $-y^2 + (1 + 3i)y + (3 + 3i)x - 1 = 0$, where $i^2 = -1$. What is the value of $x + y$?
- (A) 0
(B) -1
(C) 2
(D) 4
(E) None of the above
24. In how many different ways can the seven letters in the word MINIMUM be arranged?
- (A) 42
(B) 420
(C) 840
(D) 5040
(E) None of the above

25. Find the sum of all integer values of a for which the following circle has no x -intercept.

$$x^2 + y^2 + (a + 1)x + ay + a + 1 = 0$$

- (A) -2
- (B) 0
- (C) 2
- (D) 3
- (E) None of the above