CALIFORNIA STATE UNIVERSITY, BAKERSFIELD
MATHEMATICS FIELD DAY 2007

Individual Medley, Junior Varsity Level

There are 25 problems. You will have 50 minutes. You will only have to turn in the answer sheet – you may keep the test. Each correct answer is worth one point. Each incorrect answer will receive a one-fourth point penalty. If you believe there is an error of some sort in a particular problem, please quietly discuss it with one of the proctors.

For each of the questions, fill in the appropriate circle on the answer sheet. When the exam is over, give your answer sheet to the proctor.

Calculators of any kind are not allowed.

GOOD LUCK!

1. Least common multiple of 45 and 60 is

   (A) 90

   (B) 180

   (C) 360

   (D) 2700

   (E) None of the above

2. If $-1 < x < 3$, then $|x-3| + |x+1|$ equals

   (A) $2x - 2$

   (B) 4

   (C) $4 - x$

   (D) $2 - 2x$

   (E) None of the above

3. Two squares are drawn on a plane. If a diagonal of the first square is used as one of the sides of the second square, then the ratio of the area of the first square to that of the second one is

   (A) $\frac{1}{\sqrt{2}}$

   (B) $\frac{1}{2}$

   (C) $\sqrt{2}$

   (D) 2

   (E) None of the above

4. Suppose $f(n)$ is a function such that $f(1) = 1$, $f(2) = 2$, and $f(n+2) = f(n) + 2f(n+1)$ for all natural number n. Then $f(5)$ equals

   (A) 5

   (B) 12

   (C) 15

   (D) 23

   (E) None of the above

5. The domain of the function $f(x) = \frac{\sqrt{9-x^2}}{\sqrt{x-2}}$

   (A) $[2, 3]$

   (B) $(-\infty, 2) \cup [3, \infty)$

   (C) [-3,3]

   (D) (2,3]

   (E) None of the above

6. For nonzero real numbers a and b, the line $2x + ay + 1 = 0$ is parallel to the line $ax + by + 2 = 0$ and is perpendicular to the line $bx - y - 1 = 0$. Then, a value of ab is

   (A) 2

   (B) -1

   (C) $\frac{1}{2}$

   (D) $\frac{4}{3}$

   (E) None of the above

7. A stock gained 15% its value on Monday. On Tuesday it lost 20% of the value it had at the end of Monday. What is the overall percent loss in value from the beginning of Monday to the end of Tuesday?

   (A) 5%

   (B) 15%

   (C) 20%

   (D) 35%

   (E) None of the above

8. Kayla drove 390 miles in 8 hours. First she went for a while at 40 mi/hr and then drove the rest of the way at 60 mi/hr. How many miles did she drive at the slower speed?

   (A) 180

   (B) 200

   (C) 220

   (D) 240

   (E) None of the above

9. Under which condition is $\frac{xy}{x-y}$ negative?

(A) $0 < y < x$

(B) $y < x < 0$

(C) $x < 0 < y$

(D) $x < y < 0$

(E) None of the above

10. If $f(x) = \sqrt{x-3}$ and $g(x) = \frac{2x+1}{x-1}$, then $(g \circ f)(7)$ equals

(A) 7

(B) 5

(C) 1

(D) -2

(E) None of the above

11. If $\frac{2x-3y}{2x+y} = 7$, what is the value of $\frac{y}{x}$?

(A) 1

(B) $\frac{2}{3}$

(C) $-\frac{2}{3}$

(D) $-\frac{6}{5}$

(E) It cannot be determined.

12. The sum of all real values of x satisfying $x + 1 = \sqrt{3x+7}$ is

(A) -1

(B) 1

(C) 3

(D) 5

(E) None of the above

13. The unit digit of 7^{39} is

    (A) 1

    (B) 3

    (C) 7

    (D) 9

    (E) None of the above

14. The height of a cone is equal to the radius of its base. The radius of a sphere is equal to the radius of the base of the cone. The ratio of the volume of the cone to that of the sphere is

    (A) 1

    (B) $\frac{1}{2}$

    (C) $\frac{1}{3}$

    (D) $\frac{1}{4}$

    (E) None of the above

15. If $\left[\frac{(x^n)^3x^2}{x^n}\right]^{\frac{1}{2}} = \frac{1}{\sqrt{x}}$, then n equals

    (A) 1

    (B) $\frac{1}{2}$

    (C) $-\frac{1}{2}$

    (D) $-\frac{3}{2}$

    (E) None of the above

16. Let $9 < x < 17$. If the median and the mean of the data values $2, 4, 9, x, 17$ are equal, then the value of x is

    (A) 10

    (B) 12

    (C) 13

    (D) 15

    (E) None of the above

17. Two marbles are drawn from an urn which contains 3 yellow, 4 red, and 5 blue marbles. What is the probability that at least one of them is red?

    (A) $\frac{4}{9}$

    (B) $\frac{5}{9}$

    (C) $\frac{16}{33}$

    (D) $\frac{19}{33}$

    (E) None of the above

18. Find the area of the region where the following inequalities are satisfied: $x \leq 5$, $y \geq 1$, and $3x - 4y + 1 \geq 0$.

    (A) $\frac{5}{2}$

    (B) 5

    (C) 6

    (D) 8

    (E) None of the above

19. If $x^2 - 1$ is a factor of $2x^3 + ax^2 + bx - 1$, then $a - 2b$ equals

    (A) -1

    (B) 0

    (C) 1

    (D) 3

    (E) None of the above

20. In how many different ways can 8 people be divided into two groups, one with 3 people and the other with 5 people?

    (A) 15

    (B) 56

    (C) 336

    (D) 40320

    (E) None of the above

21. Find $f(-2)$ where the function f is a cubic function whose graph has the x-intercepts -3, -1, and 2 and the y-intercept -24.

(A) 4

(B) -6

(C) -12

(D) 16

(E) None of the above

22. It takes 6 hours for Michael to clean a house and it takes 4 hours for Michael and Kayla to clean the same house together. How long will it take for Kayla to clean the house alone?

(A) 6 hours

(B) 9 hours

(C) 12 hours

(D) 15 hours

(E) None of the above

23. A point P is randomly selected from a triangular region bounded by $(0, 0), (4, 0)$ and $(0, 4)$. What is the probability that P is at least one unit away from both of the axes?

(A) $\frac{1}{4}$

(B) $\frac{1}{2}$

(C) $\frac{3}{4}$

(D) $\frac{1}{8}$

(E) None of the above

24. Michael drove his car from City A to City B with the average speed 40 mi/hr. On his way back, he drove the car with the average speed 60 mi/hr. The average speed of Michael's car during the round trip is

(A) 48 mi/hr

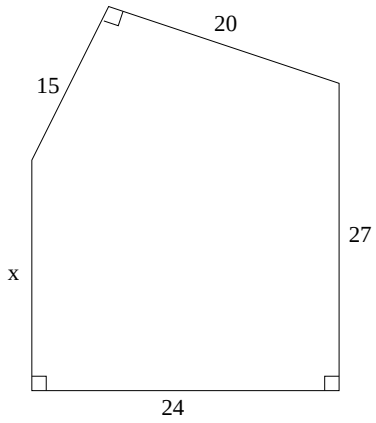
(B) 50 mi/hr

(C) 52 mi/hr

(D) It cannot be determined.

(E) None of the above

25. What is the area of the pentagon shown here with sides of length 15, 20, 27, 24, and x inches?



(A) 798 in^2

(B) 714 in^2

(C) 688 in^2

(D) 648 in^2

(E) It cannot be determined without the value of x