Surface Science 263 (1992) 680–685
North-Holland

surface science

Effect of quenched disorder on a two-dimensional classical Wigner crystal

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Received 10 June 1991; accepted for publication 26 August 1991

The effect of quenched random fields of different types on translational and orientational order and on dislocation melting of a 2D classical Wigner crystal is considered. Isotropic impurities interact only with the longitudinal plasma mode of a 2D Wigner solid and do not destroy, in the harmonic phonon approximation, the 2D translational order for potentials of the lattice-impurity interaction decaying as r^{-1} or faster. Charged Coulomb impurities embedded in a 2D crystal, if their concentration does not exceed the critical value, may induce the low-temperature reentrant melting. Ionized remote donors destroy the solid phase, unless charge fluctuations are strongly suppressed. For the substrate-generated anisotropic random pinning force there is only a short-range crystalline order, though the sizes of the crystalline regions may be arbitrarily large.

1. Introduction

Currently there is considerable interest in the Wigner crystallyzation in 2D systems. For electrons [1] or helium ions [2] at the surface of liquid helium, due to low concentrations $n \leq 10^9$ cm^{-2}, a 2D classical Wigner crystal is formed. For 2D electrons in high-quality GaAs-heterostructures, where typical concentrations $n \approx 10^{11}$ cm^{-2} by several orders exceed that in the liquid-helium systems, the Wigner crystallization can be induced in the presence of a strong magnetic field [3–5]. For this system in the quantum regime, for the lowest Landau level filling factors $\nu = nhc/eB \leq \frac{1}{5}$ there is the competition between the formation of the Wigner crystal and the quantum incompressible-Laughlin-liquid state [5]. Further increase of the magnetic field B suppresses quantum fluctuations, and for small Landau level fillings $\nu \ll 1$ the Wigner crystal in the classical regime is the ground state.

Experiments [6] and numerical simulations [7] suggest that melting of the classical Wigner crystal (CWC) is the continuous dislocation-mediated melting [8]. In this paper, we study the effect of quenched random fields on translational and orientational orders and on the dislocation melting of CWC.

Within the concept of the dislocation melting the effect of quenched random impurities [9], random topography of a substrate [10] and the substrate-generated random pinning force [11–13] were included into consideration in the case of the 2D crystal with short-range forces between particles. The case of the 2D CWC differs in two closely connected aspects. Firstly, the Wigner crystal is incompressible in the long-wavelength limit due to the long-range Coulomb forces. And, secondly, the disorder with long-range correlations (e.g., due to ionized impurities) is physically realizable and important.

The structure of the paper is as follows. In section 2 the model is introduced. We describe the CWC by the elastic-theory free energy which accounts for the incompressibility in the long-wavelength limit. Different types of quenched disorder are introduced phenomenologically: (i) quenched random impurities embedded in the CWC with different potentials of the lattice-impurity interaction $V(r)$, including long-range potentials, (ii) ionized remote donors and (iii) the substrate-generated random pinning force.

In section 3 effects due to quenched randomness are derived in the phonon approximation (thermally activated dislocations excluded) by studying the long-distance behavior of correlation functions describing the translational order (TO) and the orientational order (OO). The random fields (i) and (ii) are irrotational and, hence, interact only with the longitudinal plasma mode of the CWC. As a result, they do not affect the long-range OO and do not break the 2D power-law of the TO (for potentials $V(r)$ decaying as r^{-1} or faster). Hence, there is the possibility of a true phase transition in the system, such as melting.

The substrate-generated random fields (iii) interact with the transverse phonon mode and destroy the 2D TO, however, weak they are: the correlation function for TO decays exponentially. Hence, there is only a short-range crystalline order in the system, and the true phase transition is impossible.

Nevertheless, to understand the nature of the low-temperature phase in both cases it is important to introduce dislocations. The effect of the disorder on the break-up of dislocation pairs is briefly discussed in section 4. More detailed consideration will be presented elsewhere.

2. The model

We shall describe the 2D CWC with the triangular lattice in the continuum limit by the elastic free energy [14]

$$F_0 = \int d^2r \left[\tfrac{1}{2}\lambda u_{kk}^2 + \mu u_{ik}^2 + \tfrac{1}{2}n^2\, e^2 \int d^2r' \frac{u_{kk}(r) u_{ll}(r')}{\kappa |r - r'|} \right], \tag{1}$$

where the strain tensor u_{ik} is related to the displacement field $\boldsymbol{u}(\boldsymbol{r})$ as $u_{ik} = \frac{1}{2}(\partial u_i/\partial r_k + \partial u_k/\partial r_i)$, λ and μ are the Lamè elastic constants (which we consider as phenomenological constants), n the areal electron density, κ the effective dielectric constant. From the usual elastic theory the free energy F_0 of eq. (1) differs by the last non-local term which accounts for the contribution of long-range Coulomb forces into the free energy of static deformation causing local deviations of the charge density $\delta\rho(\boldsymbol{r}) = neu_{kk}$ [#1]. This allows one to take into account the incompressibility of the 2D CWC in the long-wavelength limit $q \to 0$ (see for the dynamical matrix eq. (11) below).

The presence of the quenched disorder is described by the part in the free energy

$$F_1 = \int d^2r \boldsymbol{f}(\boldsymbol{r}) \cdot \boldsymbol{u}(\boldsymbol{r}), \tag{2}$$

in which the areal density of the random force $\boldsymbol{f}(\boldsymbol{r})$ is coupled linearly to the displacement $\boldsymbol{u}(\boldsymbol{r})$. Different types of quenched disorder are introduced phenomenologically as follows (see also refs. [9–13]).

In the first case, the random force $\boldsymbol{f}$ is associated with the potential (or stress) field due to randomly distributed impurities for which the Gaussian distribution of concentration at long wavelengths is assumed. We consider:

(i) Quenched impurities which are embedded directly in the 2D crystal and are free to participate in the long-wavelength motion. For the discussion of such quenched randomness in the case of a 2D crystal with short-range forces see ref. [9]. For a Wigner solid we shall assume the possibility of different effective potentials of the lattice–impurity interaction $V(r)$, including long-range potentials. For the Fourier components of the random force $f_i(\boldsymbol{q})$ we have the Gaussian correlation function

$$[f_i(\boldsymbol{q}) f_k(-\boldsymbol{q})] \equiv \mathscr{T}_{ik}^{(1)}(\boldsymbol{q}) = n_{\mathrm{imp}} S_{\mathrm{imp}}(q) q_i q_k |V(\boldsymbol{q})|^2, \tag{3}$$

where $V(\boldsymbol{q})$ is the Fourier transform of the potential $V(\boldsymbol{r})$, n_{imp} is the areal density of impurities, $S_{\mathrm{imp}}(q)$ takes into account phenomenology of correlations in their positions.

[#1] Due to a misprint, the last term in F_0 is erroneously appeared in the EP2DS-9 workbook (p. 661) as a quartic anharmonic term.

(ii) Remote ionized donors in the GaAlAs layer which are distributed randomly in a volume of thickness t with a minimal separation α from 2D electrons at the GaAs/GaAlAs interface. Assuming z_0, $t \ll \alpha$, where z_0 is the penetration of electrons into the GaAs, in the Fourier representation we have (see also ref. [15])

$$\mathcal{T}_{ik}^{(2)}(\boldsymbol{q}) = n_{\mathrm{imp}} S_{\mathrm{imp}}(q) q_i q_k \left(\frac{2\pi n e^2}{\kappa q} \right)^2 \exp(-2\alpha q) . \tag{4}$$

The other type of quenched disorder is the substrate-generated random pinning force $\boldsymbol{f}(\boldsymbol{r})$ which strongly violates the invariance under the uniform translations. For such a random force we assume the Gaussian distribution

$$[f_i(\boldsymbol{r}) f_k(\boldsymbol{r}')] = \sigma\delta(\boldsymbol{r}-\boldsymbol{r}'), \quad \mathcal{T}_{ik}^{(3)}(\boldsymbol{q}) = \sigma\delta_{ik} , \tag{5}$$

where the parameter σ is an increasing function of disorder.

The important difference between the two types of quenched randomness manifests itself in the difference in the tensor structure (apart from different $\boldsymbol{q}$-behavior) of the correlation functions (3)–(5): $\mathcal{T}_{ik}^{(1)}(\boldsymbol{q})$ and $\mathcal{T}_{ik}^{(2)}(\boldsymbol{q})$ are only longitudinal with respect to vector $\boldsymbol{q}$, so that such random fields are "coupled" only to the longitudinal plasma mode of the 2D CWC, which is more rigid than the transverse phonon mode.

3. Long-range translational and orientational order in the presence of quenched random fields

Correlations functions [8] for TO and OO are given respectively by

$$C_{\mathrm{G}}(\boldsymbol{r}) = [\langle \exp \mathrm{i}\boldsymbol{G}(\boldsymbol{u}(\boldsymbol{r}) - \boldsymbol{u}(0))\rangle] , \tag{6}$$

$$C_{\mathrm{O}}(\boldsymbol{r}) = [\langle \exp \mathrm{i}6(\boldsymbol{\Theta}(\boldsymbol{r}) - \boldsymbol{\Theta}(0))\rangle] , \tag{7}$$

where the angular brackets denote thermal average with the free energy $F = F_0 + F_1$ at temperature T (we set $k_{\mathrm{B}} = 1$), and the square brackets denote subsequent average over the disorder, $\boldsymbol{G}$ is a reciprocal lattice vector, $\boldsymbol{\Theta} = \frac{1}{2}\epsilon_{ik}\, \partial_i u_k$, ϵ_{ik} being the antisymmetric tensor.

For correlation functions one obtains the multiplicative form $C_i(\boldsymbol{r}) = C_i^{(0)}(\boldsymbol{r}) C_i^{(1)}(\boldsymbol{r})$ $(i = \boldsymbol{G}, \mathrm{O})$, where $C_i^{(0)}$ is the correlation function of the pure (defect-free) system and $C_i^{(1)}$ is the part corresponding to disorder. For a pure 2D CWC in the limit $r \gg a_0$, where a_0 is the lattice spacing, we obtain the behavior

$$C_{\mathrm{G}}^{(0)}(\boldsymbol{r}) \propto r^{-\eta_{\mathrm{G}}^{(0)}}, \ \eta_{\mathrm{G}}^{(0)} = \frac{T}{4\pi\mu} G^2, \ C_{\mathrm{O}}^{(0)}(\boldsymbol{r}) \approx \exp\left\{ -\frac{9T}{4\pi\mu a_0^2} \right\} , \tag{8}$$

corresponding to the 2D power-law TO and the true long-range OO for the incompressible ($\lambda \to \infty$) 2D crystal.

For the disorder part of the correlation functions we have

$$C_{\mathrm{G}}^{(1)}(\boldsymbol{r}) = \exp\left\{ -G_i G_k \int \frac{\mathrm{d}q^2}{(2\pi)^2} D_{ij}^{-1} D_{kl}^{-1} \mathcal{T}_{jl}(\boldsymbol{q}) [1 - \cos(\boldsymbol{q}\cdot\boldsymbol{r})] \right\} , \tag{9}$$

$$C_{\mathrm{O}}^{(1)}(\boldsymbol{r}) = \exp\left\{ -9 \int \frac{\mathrm{d}q^2}{(2\pi)^2} q^2 P_{ij}^{\mathrm{tr}} D_{ik}^{-1} D_{jl}^{-1} \mathcal{T}_{kl}(\boldsymbol{q}) [1 - \cos(\boldsymbol{q}\cdot\boldsymbol{r})] \right\} , \tag{10}$$

where D_{ik}^{-1} is the inverse of the dynamical matrix D_{ik} #2 given by

$$D_{ik}(\boldsymbol{q}) = \mu q^2 P_{ik}^{\mathrm{tr}} + (2\mu + \bar{\lambda}) q^2 P_{ik}^{l} , \ \bar{\lambda} = \lambda + \frac{2\pi n^2 e^2}{\kappa q} , \tag{11}$$

and

$$P_{ik}^{\mathrm{tr}}(\boldsymbol{q}) = \delta_{ik} - q_i q_k/q^2,\ P_{ik}^{l}(\boldsymbol{q}) = q_i q_k/q^2$$

are the transverse and longitudinal projection operators.

Below we present the results for the large-r behavior of the correlation functions (9), (10) obtained in the usual harmonic phonon approximation (dislocations excluded) for different types of disorder. Note that the irrotational fields do not affect the OO.

3.1. Quenched impurities embedded in a 2D crystal

(i) For impurities with potentials $V(\boldsymbol{r})$ decaying faster than r^{-1} we obtain $C_{\mathrm{G}}^{(1)} \to$ constant < 1, $r \to \infty$. Hence, the 2D TO of the CWC is qualitatively unaffected by such impurities (compare ref. [9]). The moderate amount of disorder is obviously assumed here, so that $C_{\mathrm{G}}^{(1)}$ is not too small in comparison with unity. Otherwise, the fact that $C_{\mathrm{G}}^{(1)} \ll 1$ would be an indication of the crystal–glass transition induced by short-range impurities.

(ii) For quenched ionized impurities with the Coulomb potential of interaction $V(\boldsymbol{r}) = ne^2/\kappa r$ we obtain the algebraic decay

$$C_{\mathrm{G}}^{(1)}(\boldsymbol{r}) \propto r^{-\eta_{\mathrm{G}}^{(1)}},\ \eta_{\mathrm{G}}^{(1)} = \frac{C n_{\mathrm{imp}}}{4\pi n^2} G^2, \tag{12}$$

where the weak angular dependence is omitted, $C \leq 1$ takes account of the correlations in the impurity positions.

Hence, quenched random charged impurities embedded in a 2D CWC play the role of short-range impurities (the centers of local dilations or compressions) embedded in the 2D crystal with short-range forces considered in ref. [9]. Quenched randomness of charged impurities perhaps may be realized for the systems such as charged polystyrene spheres confined into two dimensions [16] or for 2D electrons and helium ions at the liquid helium surface. For these systems the correlations in the impurity positions may be rather important and may lead to $C \ll 1$.

(iii) We also consider long-range potentials of the lattice–impurity interaction which behave as $V(r) \propto r^{-\alpha}$, $0 < \alpha < 1$ for $r \gg a_0$, so that $V(q) \propto q^{-2+\alpha}$, $q \to 0$. In this case we obtain an exponentially decaying correlation function

$$C_{\mathrm{G}}^{(1)}(\boldsymbol{r}) \propto \exp\left\{ -B\left[\frac{G^2}{\epsilon} + \left(\boldsymbol{G} \cdot \frac{\boldsymbol{r}}{r} \right)^2 \right] r^{\epsilon} \right\},\ \epsilon = 2(1-\alpha). \tag{13}$$

Note that the exponential behavior of eq. (13) is reduced to the algebraic decay of eq. (12) for $\alpha \to 1^-$. This consideration is rather formal, however, since such random fields are too singular to be physically realizable.

#2 The dispersion relations for the transverse and longitudinal phonon modes of the defect-free 2D WC are determined in the long-wavelength limit by the transverse and longitudinal parts of D_{ik}, correspondingly, $\omega_{\mathrm{t}}^2 = \mu q^2/mn$ and $\omega_{l}^2 = 2\pi n^2 e^2 q/mn\kappa$ (m is the mass of an electron). In the presence of magnetic field, the inclusion of the Lorentz force gives the usual magnetophonon ω_- and magnetoplasma ω_+ modes (see, e.g., ref. [18]). Note, that the results for the crystalline order, in the *classical* regime, are rather insensitive to the details of the phonon spectra.

3.2. Ionized remote donors

Using eqs. (4) and (9), for $r \gg r_s$, a_0, z_0, t, where z_0, t were defined in section 2 and $r_s = \kappa(\lambda + 2\mu)/2\pi n^2 e^2$ is the screening length of the 2D CWC, we obtain

$$C_G^{(1)}(r) \approx \exp\left[-\eta_G^{(1)} \ln\left(\frac{2\alpha + \sqrt{4\alpha^2 + r^2}}{4\alpha}\right)\right], \quad \eta_G^{(1)} = \frac{Cn_{imp}}{4\pi n^2} G^2. \tag{14}$$

$C \leq 1$, as above, phenomenologically takes account of the correlations in the positions of impurities.

There are two distinct regions where $C_G^{(1)}$ has different behaviors. For $r \lesssim 2\alpha$ there is only a weak r-dependence, and here $C_G^{(1)} \approx 1$, so that the crystalline order is practically unaffected. For $r \gg 2\alpha$ we obtain power-law decay $C_G^{(1)} \approx (r/4\alpha)^{-\eta_G^{(1)}}$. Hence, in the phonon approximation, the fluctuations in the charge density of ionized remote donors do not destroy the 2D crystalline TO of the 2D CWC. Note, however, that since in this system $n_{imp} \approx n$, the exponent $\eta_G^{(1)}$ is *not small*, unless $C \ll 1$. For example, for the first reciprocal-lattice point $G_0^2 = 16\pi^2/3a_0^2$ we have $\eta_{G_0}^{(1)} = 2\pi C n_{imp}/\sqrt{3}\, n \approx 3.6 C n_{imp}/n$.

3.3. The substrate-generated random pinning force

Applying the infrared cut-off on the scale of the inverse size of the system L^{-1} to the logarithmically divergent integral in the exponent of eq. (9), we obtain for TO and OO, respectively,

$$C_G^{(1)}(r) \propto r^{-\eta_G^{(1)}} \exp\left[-\frac{\sigma G^2}{32\pi\mu^2}\left(3r_\perp^2 + r_\parallel^2\right) \ln\left(\frac{L}{r}\right)\right], \quad \eta_G^{(1)} = \frac{\sigma G^2}{16\pi^3 n^4 e^4}, \tag{15}$$

$$C_O^{(1)}(r) \propto r^{-\eta_O^{(1)}}, \quad \eta_O^{(1)} = \frac{9\sigma}{2\pi\mu^2}, \tag{16}$$

where $r_\perp$, $r_\parallel$ are perpendicular and parallel part of $\boldsymbol{r}$ with respect to vector $\boldsymbol{G}$. Hence, the fluctuations of the longitudinal plasma mode of the 2D CWC coupled to the quenched random force give an algebraically decaying contribution to the correlation function for TO, while the fluctuations of the transverse phonon mode give exponentially decaying contribution to the correlation function for TO and power-law decay of OO (compare refs. [11–13]). Therefore, there is only a short-range TO in the system.

4. Effect of quenched disorder on the dislocation melting

To study the critical behavior and the low-temperature properties, it is important to introduce dislocations. For random fields which do not destroy the 2D TO in the harmonic approximation, after applying the replica-trick, we study the effect of disorder in the multi-component sine-Gordon formulation using the Wilson's renormalization-group approach. Short-range impurities turn out to be irrelevant, while quenched Coulomb impurities for the 2D CWC play the role of short-range impurities in the 2D crystal with short-range forces [9]. In agreement with results of ref. [9], for the dimensionless disorder parameter $\bar{\sigma} = \sqrt{3}\, C n_{imp}/32n$ there is the critical value $\bar{\sigma}_c = (64\pi)^{-1}$ such that for $\bar{\sigma} > \bar{\sigma}_c$ the solid phase with algebraic decay of TO is impossible. For $\bar{\sigma} < \bar{\sigma}_c$ charged impurities shift the position of the usual dislocation melting and, also, they induce the low-temperature reentrant dislocation melting. Hence, a 2D CWC exists only in the finite temperature region.

In the case of ionized remote donors, since $n_{imp} \approx n$, we have $\bar{\sigma} > \bar{\sigma}_c$, unless the charge fluctuations are strongly suppressed (e.g., by continuous photoexcitation – see ref. [17]). Hence, the solid state is destroyed by the break-up of dislocation pairs, and the TO decays exponentially. Nevertheless, the regions of short-range crystalline order may be rather large at low temperatures and can be detected in the magneto-optical experiments.

In the case of the substrate-generated random force, the correlation function for TO, eq. (15), decays exponentially with the characteristic length $l \propto \sqrt{\sigma}$, and the true phase transition is impossible. For a moderate disorder, the system is a "Wigner glass" with 2D OO. For a small substrate disorder $l \gg n^{-1/2}$, however, the system may be considered as a finite correlation length 2D CWC and may undergo smeared dislocation melting. The analysis similar to that of ref. [13] (for 2D crystals with short-range forces) gives an indication of the low-temperature reentrant-melting behavior.

Acknowledgement

It is a pleasure to acknowledge many stimulating discussions with S.M. Apenko.

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