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Excitons in high magnetic fields in disordered two-dimensional systems: Weak-localization effects for composite neutral particles

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Quantum corrections to transport of two-dimensional (2D) excitons are considered in high magnetic fields. We find that despite total charge neutrality of these composite particles, there is no weak localization and the diffusion constant remains finite even without inelastic phase-breaking processes. This is due to time-reversal symmetry breaking by the magnetic field B. (The only exception is the case when the electron and hole components are exact $t\rightarrow -t$ counterparts; then 2D excitons are always localized in B.) Nevertheless, weak localization corrections change significantly the dependence of the diffusion constant $D(B)$, leading in high fields to much faster decrease of D with B in comparison with classical transport.

It is now well established that in disordered two-dimensional (2D) systems all states are localized no matter how weak a random potential is.[1–3] This phenomenon is universal for various wave propagation processes. Because the origin of the effect is in the constructive interference of the time-reversed scattering paths, it cannot be explained for massive particles within the framework of the classical theory. The statistics of particles does not play a crucial role in this phenomenon (see, e.g., weak localization of phonons[4] and excitons[5]).

Localization of states means that the diffusion constant as a function of frequency $D(\omega)\rightarrow 0$ when $\omega\rightarrow 0$. If one takes into account various inelastic processes such as, e.g., electron-electron interactions or interactions with phonons,[3] then $D(\omega)$ remains finite. Its static value is determined by the incoherent phase-breaking time τ_ϕ. A magnetic field leads to some new physics in weak localization: it breaks time-reversal symmetry, thus leading to a negative magnetoresistance in electron systems.[6,7] Physically, the origin of the effect is that a charged particle acquires different phases in the magnetic field moving upward and backward along a closed loop.[8] Thus the magnetic field B breaks the constructive interference, and, hence, suppresses weak localization effects.

For excitons, i.e., bound electron-hole (e-h) pairs that are totally charge neutral, the effect of B on weak localization is not obvious, and, to the best of our knowledge, has not been studied so far. The most general question here is whether time-reversal symmetry always breaks for excitons in the magnetic field. One could expect that the exciton due to its composite structure should acquire some phase in B moving along a closed loop (though the effect can be suppressed in comparison with electrons), i.e., $t\rightarrow -t$ symmetry breaks. However, it is not always the case: when e and h components are exact $t\rightarrow -t$ counterparts, there is no symmetry breaking in B for excitons, and, hence, 2D excitons should be localized. In the simplest case of spin one-half electrons and holes, $t\rightarrow -t$ symmetry is conserved for excitons in B when the masses of the two components are equal ($m_e=m_h$) and the scattering potentials are the same ($V_e=V_h$).

In a general case it should be established by concrete analysis how B suppresses weak localization for charge-neutral excitons and in which way the internal structure of these composite particles shows up. We perform such a study in the magnetic quantum limit—for strictly 2D magnetoexcitons (MX's), i.e., excitons in high magnetic fields such that $\ell_B=(\hbar c/eB)^{1/2}\ll a_B$,[9] subjected to a weak random potential; $a_B=\epsilon\hbar^2/(m_e+m_h)e^2$ is the effective exciton Bohr radius. It turns out that the magnetic field B suppresses the divergence of maximally crossed diagrams in the exciton-antiexciton channel (an analog of the cooperon[2]). As a result, the diffusion constant of 2D MX's in the presence of weak disorder remains finite in the limit $\omega\rightarrow 0$. We show that the static exciton diffusion constant $D=D(B)$ turns out to be a decreasing function of B in the limit $\ell_B\ll a_B$.

Necessarily, the effective scattering potential for excitons has always a finite correlation length due to their composite structure, even if the potentials acting on the electron and hole are δ-correlated.[5] The long-range character of the random potential together with time-reversal symmetry breaking in B leads to a new feature. We find in this case that the *transport* coefficient $\tilde{\gamma}_{\rm tr}=\hbar/\tilde{\tau}_{\rm tr}$ coming from the class of maximally crossed diagrams differs from the usual transport coefficient $\gamma_{\rm tr}=\hbar/\tau_{\rm tr}$ (coming from the class of ladder diagrams, the diffuson); here $\tau_{\rm tr}$ is the transport relaxation time. It means that in a general case two different diffusion constants—in the cooperon and in the diffuson—appear in the theory of weak localization. This fact, as far as we know, has not been established so far.

First we find the effective potential, which acts on the MX as a whole. In this paper we adopt the "exciton" approximation in which excitonic states serve as a basis (see, e.g., Ref. 13). Let us suppose that the electron and the hole are subjected to the random potentials $V_e(\mathbf{r})$ and $V_h(\mathbf{r})$, respectively. These can be, e.g., the impurity potentials or the effective potentials describing the interface roughness (IFR) in a quantum well (QW); the situation can be changed from completely statistically correlated $V_e(\mathbf{r})$ and $V_h(\mathbf{r})$ to statistically independent ones—as, e.g., for spatially separated e and h.[14] Using the wave functions of the MX in the zero Landau level,[11] we obtain the scattering matrix element be-

tween the states with momenta $\mathbf{p}$ and $\mathbf{p}'$ in an external potential $V=V_e(\mathbf{r}_e)+V_h(\mathbf{r}_h)$,

$$V_{\mathbf{p}',\mathbf{p}}=\frac{1}{S}\tilde{V}_e(\Delta\mathbf{p})\exp\left(\frac{i}{2}[\mathbf{p}'\times\mathbf{p}]_z\ell_B^2-\frac{\Delta p^2\ell_B^2}{4}\right)$$
$$+\frac{1}{S}\tilde{V}_h(\Delta\mathbf{p})\exp\left(-\frac{i}{2}[\mathbf{p}'\times\mathbf{p}]_z\ell_B^2-\frac{\Delta p^2\ell_B^2}{4}\right). \quad (1)$$

Here $\tilde{V}_\alpha(\mathbf{p})$ are 2D Fourier transforms of the potentials $V_\alpha(\mathbf{r})$ ($\alpha=e,h$), $\Delta\mathbf{p}=\mathbf{p}'-\mathbf{p}$ is the transferred momentum, and S is the area of the system. Transitions to higher Landau levels in strong magnetic fields are weak[11–13] as $(\omega_c\tau)^{-2}\sim B^{-1}\ll 1$ and irrelevant.

For a Gaussian disorder we use the standard diagram technique,[15] which involves in our case the disorder-averaged two-particle MX propagators $G_\omega^{r(a)}(p)=[\omega-\varepsilon(p)\pm i\gamma_0(p)]^{-1}$ (see, e.g., Ref. 13). The damping constant for the MX with momentum $\mathbf{p}$ is determined by the imaginary part of the self-energy [Fig. 1(a)]:

$$\gamma_0(p)=-\mathrm{Im}\int\frac{d\mathbf{p}'}{(2\pi)^2}\frac{W(\mathbf{p},\mathbf{p}',0)}{\varepsilon-\varepsilon(p')+i\gamma_0(p')}, \quad (2)$$

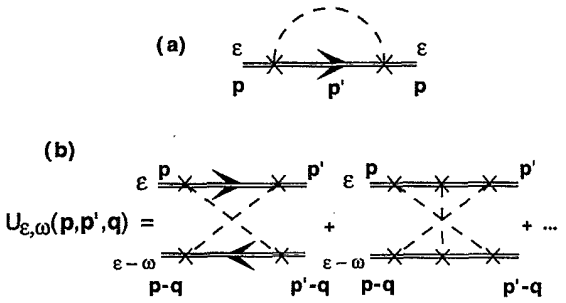

FIG. 1. (a) The lowest-order self-energy part of the MX propagator. The dashed line represents the correlation function $W(\mathbf{p},\mathbf{p}',0)$. (b) The sum of all maximally crossed diagrams, $U_{\varepsilon,\omega}(\mathbf{p},\mathbf{p}',\mathbf{q})$. The upper (lower) solid lines are disorder-averaged retarded (advanced) MX propagators $G^{r(a)}$.

where (for a parabolic part of the spectrum $p\ell_B\lesssim 1$) $\varepsilon(p)=p^2/2M$ and $M=2\hbar^2/E_0\ell_B^2\sim B^{1/2}$ is the effective mass, and $E_0=(\pi/2)^{1/2}e^2/\epsilon\ell_B\sim B^{1/2}$ is the binding energy of the 2D MX.[11] $W(\mathbf{p},\mathbf{p}_1,\mathbf{q})$ is the correlation function of the scattering potential:

$$W(\mathbf{p},\mathbf{p}_1,\mathbf{q})\equiv\langle V_{\mathbf{p},\mathbf{p}_1}V_{\mathbf{p}_1-\mathbf{q},\mathbf{p}-\mathbf{q}}\rangle=B_{ee}(\Delta\mathbf{p})\exp\{(i/2)[\mathbf{q}\times\Delta\mathbf{p}]_z\ell_B^2\}+B_{hh}(\Delta\mathbf{p})\exp\{-(i/2)[\mathbf{q}\times\Delta\mathbf{p}]_z\ell_B^2\}$$
$$+B_{eh}(\Delta\mathbf{p})\exp\{i[\mathbf{p}_1\times\mathbf{p}]_z\ell_B^2-(i/2)[\mathbf{q}\times\Delta\mathbf{p}]_z\ell_B^2\}$$
$$+B_{he}(\Delta\mathbf{p})\exp\{-i[\mathbf{p}_1\times\mathbf{p}]_z\ell_B^2+(i/2)[\mathbf{q}\times\Delta\mathbf{p}]_z\ell_B^2\}. \quad (3)$$

Here $\Delta\mathbf{p}\equiv\mathbf{p}-\mathbf{p}_1$, the correlators are defined as $B_{\alpha\beta}(\mathbf{p})=\exp(-p^2\ell_B^2/2)\langle\tilde{V}_\alpha(\mathbf{p})\tilde{V}_\beta(-\mathbf{p})\rangle$, and $\langle\cdots\rangle$ means the ensemble averaging. As usual, when $\gamma_0\ll\varepsilon$ we have

$$\gamma_0(p)=\pi\mathcal{N}(\varepsilon)\int\frac{d\phi_{p_1}}{2\pi}W(\mathbf{p},\mathbf{p}_1,0), \quad (4)$$

where $|\mathbf{p}_1|$ lies on the mass shell $\varepsilon(p_1)=\varepsilon$ and in Eq. (2) only angle average remains; $\mathcal{N}(\varepsilon)\sim B^{1/2}$ is the density of states of the 2D MX. Weak localization theory works well for a weak disorder when $\gamma_0(p)\ll\varepsilon(p)$. Formally, this condition allows one to calculate all integrals involving the Green's functions in the pole approximation. Physically, it describes the situation in which the interaction with a single scattering center cannot localize the particle and the localization is possible only at large distances due to the interference effect. For any concrete system this condition must be established separately. For MX's it is violated in the region of small p; i.e., long-wavelength MX's are strongly localized.[14] Nevertheless, when $p>p_{\min}$, the condition $\gamma(p)\ll\varepsilon(p)$ is satisfied, and the present theory is valid. For, e.g., the scattering of the MX due to the IFR in a QW of the width d, $p_{\min}\ell_B\sim\Delta\Lambda a_B/d^3$, where Δ is the mean height and Λ is the correlation length of the IFR.[14,16]

In the weak disorder limit the main contribution to the *diffusion* constant corrections comes from the class of maximally crossed diagrams in the particle-antiparticle channel.[2] For the present case of 2D MX's they are shown in Fig. 1(b). The crucial role of such diagrams is explained by the fact that when $\mathbf{p}+\mathbf{p}'-\mathbf{q}\simeq 0$, this condition holds for any total momentum in G^rG^a lines because of the momentum conservation. This means that through the diagram the poles of any pair G^r and G^a are close together (in resonance), thus leading to a maximal contribution for diagrams of the same order. It is convenient to write down the equation for the sum of such diagrams, U, in the variables $\mathbf{p}$, $\mathbf{p}'$ and the total momentum $\mathbf{K}=\mathbf{p}+\mathbf{p}'-\mathbf{q}$ ($\mathbf{q}$ is the density fluctuation momentum). For the diagrams shown in Fig. 1(b), $\mathbf{K}$ is conserved through any interaction line. Then for U in the usual way we obtain the Bethe-Salpeter equation

$$U_{\varepsilon,\omega}(\mathbf{p},\mathbf{p}',\mathbf{K})=U^0_{\varepsilon,\omega}(\mathbf{p},\mathbf{p}',\mathbf{K})$$
$$+\int\frac{d\mathbf{p}_1}{(2\pi)^2}\tilde{W}(\mathbf{p},\mathbf{p}_1,\mathbf{K})G^r_\varepsilon(\mathbf{p}_1)$$
$$\times G^a_{\varepsilon-\omega}(\mathbf{K}-\mathbf{p}_1)U_{\varepsilon,\omega}(\mathbf{p}_1,\mathbf{p}',\mathbf{K}), \quad (5)$$

where

$$U^0_{\varepsilon,\omega}(\mathbf{p},\mathbf{p}',\mathbf{K})=\int\frac{d\mathbf{p}_1}{(2\pi)^2}\tilde{W}(\mathbf{p},\mathbf{p}_1,\mathbf{K})G^r_\varepsilon(\mathbf{p}_1)$$
$$\times G^a_{\varepsilon-\omega}(\mathbf{K}-\mathbf{p}_1)\tilde{W}(\mathbf{p}_1,\mathbf{p}',\mathbf{K}), \quad (6)$$

and

$$\tilde{W}(\mathbf{p},\mathbf{p}_1,\mathbf{K})=\langle V_{\mathbf{p},\mathbf{p}_1}V_{\mathbf{K}-\mathbf{p},\mathbf{K}-\mathbf{p}_1}\rangle. \quad (7)$$

The difference of the present case from the usual theory is in the difference between the correlation functions $\tilde{W}$ in Eq. (7) and W in Eq. (3). In $\tilde{W}$, in comparison with W, the phase factors are exchanged $B_{ee}\leftrightarrow B_{eh}$ and $B_{he}\leftrightarrow B_{hh}$. For example, into $\tilde{W}(\mathbf{p},\mathbf{p}_1,\mathbf{K})$ the term

$$B_{ee}(\Delta\mathbf{p})\exp\{i[\mathbf{p}_1\times\mathbf{p}]_z\,\ell_B^2+(i/2)[\mathbf{K}\times\Delta\mathbf{p}]_z\,\ell_B^2\} \quad (8)$$

enters, while into Eq. (3) the same phase factor comes with B_{eh}. This difference is connected with time-reversal symmetry breaking for the effective potential: $V_{\mathbf{p},\mathbf{p}_1}\neq V_{-\mathbf{p}_1,-\mathbf{p}}$. Otherwise, we would come to the usual theory of weak localization.

In the weak disorder limit $G^rG^a\sim\delta(\varepsilon(p)-\varepsilon)$, so only angle integrations remain in Eq. (5). Usually, the isotropic part of $U_{\varepsilon,\omega}(\mathbf{p},\mathbf{p}',\mathbf{K})$ diverges when $\mathbf{K},\omega\to 0$. This happens because the relation

$$\int\frac{d\mathbf{p}_1}{(2\pi)^2}\,W(\mathbf{p},\mathbf{p}_1,0)G_\varepsilon^r(\mathbf{p}_1)G_\varepsilon^a(-\mathbf{p}_1)=1 \quad (9)$$

holds. Then from Eqs. (5) and (9) (if $\tilde{W}=W$) it follows that $\int d\phi_p\int d\phi_{p1}\,U_{\varepsilon,\omega}(\mathbf{p},\mathbf{p}_1,0)\to\infty$. In our case, however, the isotropic part of U remains finite in the limit $\mathbf{K},\omega\to 0$. Indeed, using the identity $\tilde{W}\equiv W+(\tilde{W}-W)$, we obtain

$$\int\frac{d\mathbf{p}_1}{(2\pi)^2}\,\tilde{W}(\mathbf{p},\mathbf{p}_1,0)G_\varepsilon^r(\mathbf{p}_1)G_\varepsilon^a(-\mathbf{p}_1)=1-\frac{\gamma_B}{\gamma_0}, \quad (10)$$

where $\gamma_B(p)=\gamma_0(p)-\tilde{\gamma}_0(p)\geq 0$,

$$\tilde{\gamma}_0(p)=\pi\mathcal{N}(\varepsilon)\int\frac{d\phi_{p1}}{2\pi}\,\tilde{W}(\mathbf{p},\mathbf{p}_1,0). \quad (11)$$

Still, when $\gamma_B\ll\gamma_0$, the isotropic part of U is the leading one. The solution of Eq. (5) can be found for small ω and $\mathbf{K}$ as an expansion in angular momentum (a similar procedure was used in Ref. 17). We expand the product G^rG^a up to the second order in $\mathbf{K}$:

$$G_\varepsilon^r(\mathbf{p})G_{\varepsilon-\omega}^a(\mathbf{K}-\mathbf{p})\simeq\frac{\mathrm{Im}\,G_\varepsilon^a(\mathbf{p})}{\gamma_0(p)}\left[1-\frac{\omega-(\mathbf{p}\cdot\mathbf{K})/M}{2i\gamma_0(p)}-\frac{(\mathbf{p}\cdot\mathbf{K})^2}{M^2}\frac{1}{[2\gamma_0(p)]^2}\right]. \quad (12)$$

[For the nonparabolic part of the spectrum, $\mathbf{p}/M$ should be replaced by the exciton center-of-mass velocity $\mathbf{V}(\mathbf{p})=\partial\varepsilon_p/\partial\mathbf{p}$.] Higher angular terms are connected with the isotropic part by the term $(\mathbf{p}\cdot\mathbf{K})$, which means that every next angular term is in the following order in K and can be neglected. Also, we can set $\mathbf{K}=0$ in $\tilde{W}(\mathbf{p},\mathbf{p}_1,\mathbf{K})$ because in the $\mathbf{K}$ expansion the terms coming from $\tilde{W}$ would give an additional small factor $[\gamma(p)/\varepsilon(p)](p\ell_B)^2$. Substituting Eq. (12) into (5) and integrating on the mass shell $\varepsilon(p)=\varepsilon(p')=\varepsilon$ over the angular variables ϕ_p, ϕ_{p1}, we obtain the first equation

$$U(\mathbf{K},\omega)=U_0+\left[\left(1+\frac{i\omega}{2\gamma_0}\right)\frac{\tilde{\gamma}_0}{\gamma_0}-K^2\frac{p^2}{2M^2}\frac{\tilde{\gamma}_0}{4\gamma_0^3}\right]U(\mathbf{K},\omega)-K\,(\tilde{\gamma}_0/2\gamma_0^2)U_1(\mathbf{K},\omega), \quad (13)$$

where we omit for brevity the dependence on ε, and

$$U_0=\frac{\tilde{\gamma}_0}{\gamma_0}\int\frac{d\phi_{p1}}{2\pi}\,\tilde{W}(\mathbf{p},\mathbf{p}_1,0)=\frac{\tilde{\gamma}_0^2}{\pi\mathcal{N}(\varepsilon)\gamma_0}, \quad (14)$$

$$U(\mathbf{K},\omega)=\int\frac{d\phi_p}{2\pi}\int\frac{d\phi_{p1}}{2\pi}\,U_{\varepsilon,\omega}(\mathbf{p},\mathbf{p}_1,\mathbf{K}), \quad (15)$$

$$U_1(\mathbf{K},\omega)=i\int\frac{d\phi_p}{2\pi}\int\frac{d\phi_{p1}}{2\pi}\frac{\mathbf{p}\cdot\hat{\mathbf{K}}}{M}\,U_{\varepsilon,\omega}(\mathbf{p},\mathbf{p}_1,\mathbf{K}), \quad (16)$$

$\hat{\mathbf{K}}=\mathbf{K}/|\mathbf{K}|$. Multiplying both sides of Eq. (13) by $i(\mathbf{p}\cdot\mathbf{K})/M$ and integrating again over the directions of $\mathbf{p}$ and $\mathbf{p}_1$, we obtain the second relation

$$U_1(\mathbf{K},\omega)=K\frac{p^2}{2M^2}\frac{\tilde{\gamma}_1}{2\gamma_0^2}U(\mathbf{K},\omega)+\left(1+\frac{i\omega}{2\gamma_0}\right)\frac{\tilde{\gamma}_1}{\gamma_0}\,U_1(\mathbf{K},\omega). \quad (17)$$

Here a value $\tilde{\gamma}_1$ appears that leads from γ_0 to the transport coefficient $\tilde{\gamma}_{\mathrm{tr}}$ [see Eq. (19) below]:

$$\tilde{\gamma}_1=2\int\frac{d\phi_p}{2\pi}\int\frac{d\phi_{p1}}{2\pi}\,\hat{\mathbf{p}}\cdot\hat{\mathbf{K}}\,\tilde{W}(\mathbf{p},\mathbf{p}_1,0)\,\hat{\mathbf{p}}_1\cdot\hat{\mathbf{K}}, \quad (18)$$

where $\hat{\mathbf{p}}=\mathbf{p}/|\mathbf{p}|$. Solving the system of Eqs. (13) and (17), we finally get the cooperon vertex

$$U(\mathbf{K},\omega)=\frac{2\tilde{\gamma}_0\gamma_0/\pi\mathcal{N}(\varepsilon)}{D^cK^2-i\omega+2\gamma_B\gamma_0/\tilde{\gamma}_0}, \quad (19)$$

where $D^c=p^2/4M^2\tilde{\gamma}_{\mathrm{tr}}$, $\tilde{\gamma}_{\mathrm{tr}}=\gamma_0-\tilde{\gamma}_1\geq 0$. A notable feature presented by this equation is that for a neutral composite particle in B a finite value γ_B appears which cuts off the singularity of the cooperon (cf. Refs. 6 and 7). The value of γ_B can be estimated from Eqs. (10) and (11):

$$\gamma_B(p)\simeq\eta(p\ell_B)^4\gamma_0(p), \quad (20)$$

where $\eta=1-(\langle V_e^*V_h\rangle+\langle V_eV_h^*\rangle)/(\langle|V_e|^2\rangle+\langle|V_h|^2\rangle)$.[18] At a fixed momentum p, for IFR (Ref. 14) $\gamma_0(p)\sim B^{1/2}$, and we have $\gamma_B\sim B^{-3/2}$; in the case of unscreened charged impurities $\gamma_0(p)\sim B^{-3/2}$ and $\gamma_B\sim B^{-7/2}$. In general, with increasing B the internal structure of the MX is revealed less, and the MX more closely resembles an ordinary neutral particle. Qualitatively, we can connect the appearance of γ_B with the composite structure of the MX by the following geometric interpretation (cf. Ref. 8). For the MX the magnetic momentum $\mathbf{p}$ determines the mean separation between the electron and hole $\mathbf{r}_{eh}=\mathbf{p}\times\hat{\mathbf{z}}\,\ell_B^2$.[10,11] Then, since near the scatterer the electron and hole move *differently*,[18] there appears a random phase shift $\sim r_{eh}^2/\ell_B^2$. It is equal to the ratio of the magnetic flux that the exciton acquires, $\Delta\Phi\sim r_{eh}^2B$, to the flux quantum Φ_0. The value $\Delta\Phi/\Phi_0$ determines the random phase fluctuation for the exciton in a single act of scattering, i.e., during the relaxation time τ. The destruction of interference occurs during the time τ_B when the phase fluctuations

~ 1. Since changes of the phase are random, this is achieved over $\sim(\Phi_0/\Delta\Phi)^2$ steps, and, hence, requires the time $\tau_B \simeq (\Phi_0/\Delta\Phi)^2\tau \sim \tau/(p\ell_B)^4$, which is consistent with Eq. (20). Note that the effect of phase destruction turns out to be proportional to the fourth power of the exciton size r_{eh}, i.e., strongly *suppressed* with increasing B.

The second feature of Eq. (19) is that $\tilde{\gamma}_{\rm tr}=\gamma_0-\tilde{\gamma}_1$ differs from the usual transport coefficient $\gamma_{\rm tr}$. Indeed, the two contributions to $\tilde{\gamma}_{\rm tr}$ are determined by the isotropic part of the vertex W and the angle-dependent part of $\tilde{W}$, correspondingly. In $\gamma_{\rm tr}$ both parts are determined by W. Therefore, in this case D^c in Eq. (19) does not coincide with the "classical" diffusion constant D_0. This finding that several relaxation times appear in the theory can be addressed to any disordered system in which, simultaneously, time-reversal symmetry is broken and a random potential has a finite correlation length.

To obtain quantum corrections to the diffusion constant, the representation (19) for $U(\mathbf{K},\omega)$ together with the first-order vertex $W(\mathbf{p},\mathbf{p}',\mathbf{K})$ should be inserted into the usual ladder diagrams for the effective conductivity.[3] The solution of the ladder equations (the details will be presented elsewhere[16]) leads to the appearance of the transport coefficient $\gamma_{\rm tr}$ instead of γ, for a finite-range correlated potential.[19] Finally, the diffusion constant of the MX with energy ε is

$$D(\varepsilon)=D_0(\varepsilon)\left[1-\frac{\tilde{\gamma}_0}{4\pi^2\gamma_{\rm tr}\mathcal{N}_\varepsilon D^c}\ln\left(\frac{D^cK_0^2\tilde{\gamma}_0}{2\gamma_B\gamma_0}\right)\right], \quad (21)$$

where the cutoff momentum $K_0\simeq\gamma(p)/V(p)$ is determined by the applied approximation [see Eq. (12)] and $D_0=p^2/4M^2\gamma_{\rm tr}$ is the usual "classical" diffusion constant of the MX.[14,16] At a fixed energy ε we have $K_0\sim B^{1/4}$, and for IFR D_0, $D^c\sim B^{-1}$. In the main order in B $\tilde{\gamma}_0/\gamma_0=\tilde{\gamma}_0/\gamma_{\rm tr}=O(1)$. Thus, when B increases, the fast decay of γ_B outweighs other factors under the logarithm in Eq. (21), and the negative correction term behaves as $\sim -B^{1/2}\ln B$. For any finite energy ε at sufficiently high B the correction term becomes of the order of unity. In such a situation D can be obtained, e.g., by a self-consistent approach similar to Ref. 20, which gives $D(\varepsilon)\sim B^{-2}$. This behavior shows that quantum corrections are important at high magnetic fields and lead to a faster power-law decrease of D as compared to the classical diffusion constant $D_0\sim B^{-1}$. Note also that Eq. (20) shows that the phase-breaking effects turn out to be suppressed with increasing B, namely, $\gamma_B/\gamma_0\sim\ell_B^4$. Thus, in the limit $B\to\infty$ MX's can be considered as neutral, almost structureless quasiparticles, and for neutral e-h systems the usual effects of 2D weak localization should be recovered.

Interestingly, since at $B=0$ the static diffusion constant $D=0$ (2D excitons,[5] as well as ordinary 2D particles are localized in a random potential), and at high fields, as it was shown, D is finite and decreasing, we conclude that D should be a nonmonotonous function of B, being an *increasing* function of B at low (and possibly intermediate) fields. The latter effect for excitons can be thought of as an analog of negative magnetoresistance in electron systems. This prediction of theory can be experimentally tested at low temperatures (when the phase-breaking time τ_ϕ is large) in QW's with weak or moderate disorder (e.g., wide QW's with smooth interfaces) in magnetic fields $\ell_B \gtrsim a_B$.

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[18] When $V_e=V_h$ (and the random potentials are fully statistically correlated), in the strong magnetic field limit we have $\eta=0$ and, thus, $\gamma_B=0$ [cf. with the hidden symmetry in the disorder-free case (Ref. 12)]. Virtual transitions to higher Landau levels would give a small but finite contribution to γ_B. If, however, there is a complete $t\to -t$ symmetry between e and h ($V_e=V_h$ and $m_e=m_h$), time-reversal symmetry is conserved for the exciton and $\gamma_B\equiv 0$ at any finite B.

[19] Note that, as a by-product, we corrected the result for the cooperon derived (Ref. 4) for long-range correlated potentials [and used (Ref. 5) for excitons]: we show that it is the *transport* relaxation time which enters the diffusion constant in the cooperon—not the bare one as in Refs. 4 and 5.

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