PERGAMON

Solid State Communications 113 (2000) 683–687

solid state communications

www.elsevier.com/locate/ssc

Two-dimensional charged electron–hole complexes in magnetic fields: keeping magnetic translations preserved

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Received 3 December 1999; accepted 8 December 1999 by L.V. Keldysh

Abstract

Eigenstates of two-dimensional charged electron–hole complexes in magnetic fields are considered. The operator formalism that allows one to partially separate the center-of-mass motion from internal degrees of freedom is presented. The scheme using magnetic translations is developed for calculating in strong magnetic fields the eigenspectra of negatively charged excitons X^-, a bound state of two electrons and one hole.

Keywords: A. Semiconductors; A. Quantum wells; D. Electron–electron interactions

1. Introduction

Identification of charged excitons in magneto-optical spectra of quasi-two-dimensional (quasi-2D) systems (see, e.g. [1–6] and references therein) has induced much interest in the behavior of these three-particle electron–hole (e–h) complexes. The negatively, X^-, and positively, X^+, charged excitons are the bound states of two electrons and one hole (2e–h) and two holes and one electron (2h–e), respectively. In magnetic fields B, in addition to the spin-singlet, higher-lying triplet states of X^- and X^+ have been observed [1–6]. Theoretically, free charged excitons have been studied in strictly 2D systems in the limit of high [7] and low [8] magnetic fields and in quasi-2D systems at high magnetic fields [9,10]. For one-component electron systems in magnetic fields, the center-of-mass motion separates from internal degrees of freedom. The well-known Kohn theorem [11], which states that the electron cyclotron resonance is not shifted or broadened by electron–electron interactions, is based on this fact. For e–h systems such a complete separation is not possible in magnetic fields. Nonetheless, any charged interacting system in a uniform B possesses an exact dynamical symmetry—magnetic translations ([12,13] and references therein). It has been shown recently [14] that due to this symmetry, magneto-optical transitions of charged e–h complexes are governed by an exact selection rule, which leads to some rather unexpected spectroscopic consequences for charged excitons in B. In this work, using an operator formalism, we construct a basis compatible with the exact dynamical symmetries— rotations about the B-axis and magnetic translations. Physically, this is equivalent to a partial separation of the center-of-mass motion from internal degrees of freedom in B [12,13]. We demonstrate that this basis can be used for high-accuracy and rapidly convergent calculations of bound X^- states in strong magnetic fields. Our results can also be relevant for atomic ions with not too large mass ratios in ultrastrong magnetic fields [13].

2. Basis compatible with magnetic translations

We consider a strictly 2D system containing two electrons and one hole in a perpendicular magnetic field $\mathbf{B} = (0, 0, B)$ described by the Hamiltonian

$$H = H_0 + H_{\text{ee}} + H_{\text{eh}} \tag{1}$$

$$H_0 = \sum_{i=1,2} \frac{\hat{\boldsymbol{\pi}}_{ei}^2}{2m_e} + \frac{\hat{\boldsymbol{\pi}}_h^2}{2m_h} \tag{2}$$

$$H_{\text{ee}} = \frac{e^2}{\epsilon|\mathbf{r}_1 - \mathbf{r}_2|}, H_{\text{eh}} = -\sum_{i=1,2} \frac{e^2}{\epsilon|\mathbf{r}_i - \mathbf{r}_h|}, \tag{3}$$

where $\hat{\boldsymbol{\pi}}_j = -i\hbar\boldsymbol{\nabla}_j - \frac{e_j}{c}\mathbf{A}(\mathbf{r}_j)$ are kinematic momentum operators. We will use the symmetric gauge $\mathbf{A} = \frac{1}{2}\mathbf{B}\times\mathbf{r}$.

PII: S0038-1098(99)00556-6

The exact eigenstates can be characterized by the total angular momentum projection M_z, an eigenvalue of $\hat{L}_z = \sum_j (\mathbf{r}_j \times -i\hbar\boldsymbol{\nabla}_j)_z$, by the total spin of two electrons $S_e = 0$ (singlet states) or $S_e = 1$ (triplet states), and the spin state of the hole S_h. The latter simply factors out and will be disregarded. Performing an orthogonal transformation of the coordinates $\{\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_h\} \to \{\mathbf{r}, \mathbf{R}, \mathbf{r}_h\}$, where $\mathbf{r} = (\mathbf{r}_1 - \mathbf{r}_2)/\sqrt{2}$ is the electron relative and $\mathbf{R} = (\mathbf{r}_1 + \mathbf{r}_2)/\sqrt{2}$ center-of-mass coordinates, the complete orthonormal basis with a fixed value of M_z can be constructed [15] (see also [16]) as an expansion in Landau levels (LLs)

$$\phi^{(e)}_{n_1m_1}(\mathbf{r})\,\phi^{(e)}_{n_2m_2}(\mathbf{R})\,\phi^{(h)}_{n_hm_h}(\mathbf{r}_h)\,. \tag{4}$$

Here $\phi^{(e)}_{nm}(\mathbf{r}) = \phi^{(h)*}_{nm}(\mathbf{r})$ are the e- and h- single-particle factored wave functions in B; n is the LL quantum number and m is the oscillator quantum number (see, e.g. [12,13]). For, e.g. zero LLs

$$\phi^{(e)*}_{0m}(\mathbf{r}) = \phi^{(h)}_{0m}(\mathbf{r}) = \frac{1}{(2\pi m!\ell_B^2)^{1/2}}\left(\frac{z}{\sqrt{2}\ell_B}\right)^m \exp\left(-\frac{r^2}{4\ell_B^2}\right), \tag{5}$$

where $z = x + iy$ is the 2D complex coordinate and $\ell_B = (\hbar c/eB)^{1/2}$. The factored wave functions are constructed with the help of the oscillator Bose ladder operators: For electrons (the charge $-e < 0$)

$$\phi^{(e)}_{nm}(\mathbf{r}) = \frac{1}{\sqrt{n!m!}}\langle \mathbf{r}|(A_e^\dagger)^n (B_e^\dagger)^m|0\rangle\,, \tag{6}$$

here the intra-LL operators $B_e^\dagger(\mathbf{r}_j) = -i\sqrt{c/2\hbar Be}\,\hat{K}_{j-}$, where $\hat{K}_{j\pm} = \hat{K}_{jx} \pm i\hat{K}_{jy}$ and $\hat{\mathbf{K}}_j = \hat{\boldsymbol{\pi}}_j - \frac{e_j}{c}\mathbf{r}_j \times \mathbf{B}$ (see, e.g., [12,13]). The electron inter-LL operators are $A_e^\dagger(\mathbf{r}_j) = -i\sqrt{c/2\hbar Be}\,\hat{\pi}_{j+}$, where $\hat{\pi}_{j\pm} = \hat{\pi}_{jx} \pm i\hat{\pi}_{jy}$. The operators commute as $[A_e, A_e^\dagger] = 1$, $[B_e, B_e^\dagger] = 1$, and $[A_e, B_e^\dagger] = [A_e, B_e] = 0$. The analogous intra-LL and inter-LL operators for the hole (the charge $e > 0$) are, respectively, $B_h^\dagger(\mathbf{r}_h) = -i\sqrt{c/2\hbar Be}\,\hat{K}_{h+}$ and $A_h^\dagger(\mathbf{r}_h) = -i\sqrt{c/2\hbar Be}\,\hat{\pi}_{h-}$. These can be considered as linear functions of spatial coordinates and derivatives and have the form

$$A_e^\dagger(\mathbf{r}) = B_h^\dagger(\mathbf{r}) = \frac{1}{\sqrt{2}}\left(\frac{z}{2\ell_B} - 2\ell_B\frac{\partial}{\partial z^*}\right), \tag{7}$$

$$B_e^\dagger(\mathbf{r}) = A_h^\dagger(\mathbf{r}) = \frac{1}{\sqrt{2}}\left(\frac{z^*}{2\ell_B} - 2\ell_B\frac{\partial}{\partial z}\right). \tag{8}$$

Single-particle angular momentum projection operators $\hat{L}_{ze} = A_e^\dagger A_e - B_e^\dagger B_e$ and $\hat{L}_{zh} = B_h^\dagger B_h - A_h^\dagger A_h$, so that $m_{ze} = -m_{zh} = n - m$. The basis (4) includes therefore different three-particle $2e$–h states such that $M_z = n_1 + n_2 - m_1 - m_2 - n_h + m_h$ is fixed. Permutational symmetry of identical particles requires that for electrons in the spin-singlet $S_e = 0$ (triplet $S_e = 1$) state the relative motion angular momentum $n_1 - m_1$ should be even (odd). The basis (4) proved to be effective in strong B for studying impurity-bound states of e–h complexes [15], collective excitations—magnetoplasmons and spin-waves [17], and effects of lateral confinement in quantum dots in B [18,19]. The equivalent LL expansion (using the coordinates $\{\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_h\}$) has been exploited [9,10] for studying *free* charged excitons in B. However, for translationally invariant systems the basis (4) is *not compatible* with the magnetic translations.

Indeed, the Hamiltonian (1) commutes with the operator of the magnetic translations $\hat{\mathbf{K}} = \sum_j \hat{\mathbf{K}}_j$ [12,13]. Noting that $[\hat{K}_x, \hat{K}_y] = -i\frac{\hbar B}{c}Q$, where the total charge $Q \equiv \sum_j e_j = -e$ for the X^-, one obtains the lowering and raising Bose ladder operators for the *whole system* [12–14]

$$\hat{k}_\pm = \pm\frac{i}{\sqrt{2}}(\hat{k}_x \pm i\hat{k}_y), \qquad [\hat{k}_+, \hat{k}_-] = -\frac{Q}{|Q|} = 1, \tag{9}$$

here $\hat{\mathbf{k}} = \sqrt{c/\hbar B|Q|}\,\hat{\mathbf{K}}$. Therefore, $\hat{\mathbf{k}}^2 = \hat{k}_+\hat{k}_- + \hat{k}_-\hat{k}_+$ has the discrete oscillator eigenvalues $2k+1$, $k = 0, 1, \ldots$. These can be used, together with M_z, for labelling of exact charged eigenstates of (1). Due to the non-commutativity of $\hat{K}_x$ and $\hat{K}_y$, there is the macroscopic Landau degeneracy in k. Note now that $\hat{\mathbf{k}}^2 = \left(\sum_j \hat{\mathbf{k}}_j\right)^2 = \sum_j \hat{\mathbf{k}}_j^2 + \sum_{i\neq j}\hat{\mathbf{k}}_i\cdot\hat{\mathbf{k}}_j$ is *not diagonal* in the basis (4) due to the cross terms $\sum_{i\neq j}\hat{\mathbf{k}}_i\cdot\hat{\mathbf{k}}_j$.

In order to make the basis (4) compatible with the magnetic translations, a canonical transformation diagonalizing $\hat{\mathbf{k}}^2$ should be performed. We deal formally with a set of coupled harmonic oscillators. Note then that

$$\hat{k}_- = B_e^\dagger(\mathbf{r}_1) + B_e^\dagger(\mathbf{r}_2) - B_h(\mathbf{r}_h) = \sqrt{2}\,B_e^\dagger(\mathbf{R}) - B_h(\mathbf{r}_h) \tag{10}$$

and define

$$\tilde{B}_e^\dagger(\mathbf{R}) = uB_e^\dagger(\mathbf{R}) - vB_h(\mathbf{r}_h), \quad \tilde{B}_e(\mathbf{R}) = uB_e(\mathbf{R}) - vB_h^\dagger(\mathbf{r}_h), \tag{11}$$

where $u = \sqrt{2}$, $v = 1$. It is this pair of Bose ladder operators in which $\hat{\mathbf{k}}^2$ is diagonal: $\hat{\mathbf{k}}^2 = 2\tilde{B}_e^\dagger\tilde{B}_e + 1$. Eq. (11) is in fact a Bogoliubov canonical transformation $\tilde{B}_e^\dagger = SB_e^\dagger S^\dagger$ generated by the unitary operator (see, e.g. [20,21])

$$S = \exp\{\Theta[B_e(\mathbf{R})B_h(\mathbf{r}_h) - B_h^\dagger(\mathbf{r}_h)B_e^\dagger(\mathbf{R})]\} \tag{12}$$

with $u = \mathrm{ch}\,\Theta = \sqrt{2}$, $v = \mathrm{sh}\,\Theta = 1$. The second pair of linearly independent transformed operators $\tilde{B}_h^\dagger(\mathbf{r}_h) = SB_h^\dagger(\mathbf{r}_h)S^\dagger$ and $\tilde{B}_h(\mathbf{r}_h) = SB_h(\mathbf{r}_h)S^\dagger$ are

$$\tilde{B}_h^\dagger(\mathbf{r}_h) = uB_h^\dagger(\mathbf{r}_h) - vB_e(\mathbf{R}), \quad \tilde{B}_h(\mathbf{r}_h) = uB_h(\mathbf{r}_h) - vB_e^\dagger(\mathbf{R}). \tag{13}$$

The complete orthogonal basis compatible with both axial and translational symmetries therefore is

$$A_e^\dagger(\mathbf{r})^{n_1}A_e^\dagger(\mathbf{R})^{n_2}A_h^\dagger(\mathbf{r}_h)^{n_h}\tilde{B}_e^\dagger(\mathbf{R})^k B_e^\dagger(\mathbf{r})^m \tilde{B}_h^\dagger(\mathbf{r}_h)^l|\tilde{0}\rangle\,. \tag{14}$$

In (14) the oscillator quantum number is fixed and equals k while $M_z = -k - m + l + n_1 + n_2 - n_h$ and $n_1 - m$ is even (odd) for $S_e = 0$ ($S_e = 1$). The Hamiltonian (1) is block-diagonal in the quantum numbers k, M_z, S_e. Moreover, due to the Landau degeneracy in k, it is sufficient to consider the $k = 0$ states only. This effectively removes one degree of freedom and corresponds to a partial separation of the center-of-mass motion from internal degrees of freedom for a charged e–h system in a magnetic field (cf. [12,13]).

In (14) the new vacuum $|\tilde{0}\rangle = S|0\rangle$ has been introduced. Disentangling the operators in the exponent of S (see, e.g. [20,21]), one obtains

$$S = \exp\Big(-\mathrm{th}\,\Theta\, B_h^\dagger B_e^\dagger\Big)\exp\Big(-\ln(\mathrm{ch}\,\Theta)[B_e^\dagger B_e + B_h^\dagger B_h + 1]\Big) \times \exp(\mathrm{th}\,\Theta\, B_e B_h), \tag{15}$$

so that

$$|\tilde{0}\rangle = S|0\rangle = \frac{1}{\mathrm{ch}\,\Theta}\exp\Big(-\mathrm{th}\,\Theta\, B_h^\dagger(\mathbf{r}_h)B_e^\dagger(\mathbf{R})\Big)|0\rangle\,. \tag{16}$$

For a charged system of N_e electrons and N_h holes (with, e.g. $N_e > N_h$), a transformation analogous to (11)–(13) can also be performed. It should involve the intra-LL e- and h- center-of-mass operators $B_e^\dagger(\mathbf{R}_e)$ and $B_h(\mathbf{R}_h)$ with $\mathrm{th}\,\Theta = \sqrt{N_h/N_e}$; here $\mathbf{R}_e = \sum_{i=1}^{N_e}\mathbf{r}_{ei}/\sqrt{N_e}$ and $\mathbf{R}_h = \sum_{j=1}^{N_h}\mathbf{r}_{hj}/\sqrt{N_h}$.

3. X^- states in lowest Landau levels

We now demonstrate how the developed formalism works. We will consider the limit of high magnetic fields [7,10,14]

$$\hbar\omega_{\mathrm{ce}}\,,\ \hbar\omega_{\mathrm{ch}}\,,\ |\hbar\omega_{\mathrm{ce}} - \hbar\omega_{\mathrm{ch}}| \gg E_0 = \sqrt{\frac{\pi}{2}}\,\frac{e^2}{\epsilon l_B}\,, \tag{17}$$

when mixing between different LL's can be neglected. E_0 is the characteristic energy of the Coulomb interactions in strong B, $\hbar\omega_{\mathrm{ce(h)}} = \hbar eB/m_{\mathrm{e(h)}}c$. Charged magnetoexcitons can then be labeled by the total electron LL number $n_e = n_1 + n_2$ and by the hole LL number n_h. Indeed, when (17) is fulfilled, the states having different quantum numbers $n_e n_h$ and $n_e' n_h'$ are only weakly $\sim E_0/|(n_e' - n_e)\hbar\omega_{\mathrm{ce}} + (n_h' - n_h)\hbar\omega_{\mathrm{ch}}|$ mixed by the Coulomb interactions [16].

We focus on the states in zero LL's [$n_1 = n_2 = n_h = 0$ in (14)]. The operators (11), (13) have a simple representation in the new coordinates $\boldsymbol{\rho}_1 = \sqrt{2}\,\mathbf{R} - \mathbf{r}_h$ and $\boldsymbol{\rho}_2 = \sqrt{2}\,\mathbf{r}_h - \mathbf{R}$: $\tilde{B}_e^\dagger(\mathbf{R}) = B_e^\dagger(\boldsymbol{\rho}_1)$ and $\tilde{B}_h^\dagger(\mathbf{r}_h) = B_h^\dagger(\boldsymbol{\rho}_2)$. The complete infinite orthonormal basis in zero LL's with fixed $k = 0$ and arbitrary $M_z = l - m$ takes the form

$$\frac{1}{(m!l!)^{1/2}}B_e^\dagger(\mathbf{r})^m B_h^\dagger(\boldsymbol{\rho}_2)^l|\tilde{0}\rangle \equiv |ml\rangle \tag{18}$$

with odd $m = 2p + 1$ (even $m = 2p$), $p = 0, 1, \ldots$ in the electron triplet $S_e = 1$ (singlet $S_e = 0$) states. The Coulomb interactions in the new variables are

$$H_{\mathrm{ee}} = \frac{e^2}{\sqrt{2}\epsilon r}\quad,\qquad H_{\mathrm{eh}} = -\frac{\sqrt{2}e^2}{\epsilon|\boldsymbol{\rho}_2 - \mathbf{r}|} - \frac{\sqrt{2}e^2}{\epsilon|\boldsymbol{\rho}_2 + \mathbf{r}|}\,. \tag{19}$$

The matrix elements of the e–e interaction are diagonal in the basis (18):

$$\langle m_2 l_2|H_{\mathrm{ee}}|m_1 l_1\rangle = \delta_{m_1,m_2}\delta_{l_1,l_2}\frac{V_{0,m_1}}{\sqrt{2}}\,,\qquad V_{0,m} = \frac{(2m-1)!!}{2^m m!}\,E_0\,, \tag{20}$$

where $V_{0,m}$ is the interaction of the electron with a fixed negative charge $-e$ in zero LL (e.g. [15]). Due to the permutational symmetry, the two terms in H_{eh} give the same contributions; calculations, however, are not so straightforward as (20). This is connected with the fact that the coordinate transformation $\{\mathbf{r}, \mathbf{R}, \mathbf{r}_h\} \to \{\mathbf{r}, \boldsymbol{\rho}_1, \boldsymbol{\rho}_2\}$ is *not orthogonal*. As a result, the coordinate representation of the new vacuum is not factored in $\boldsymbol{\rho}_1$ and $\boldsymbol{\rho}_2$:

$$\langle \mathbf{r}\boldsymbol{\rho}_1\boldsymbol{\rho}_2|\tilde{0}\rangle = \frac{1}{\sqrt{2}\,(2\pi\ell_B^2)^{3/2}}\exp\left(-\frac{r^2 + \rho_1^2 + \rho_2^2 + \sqrt{2}Z_1 Z_2^*}{4\ell_B^2}\right), \tag{21}$$

here $Z_j = \rho_{jx} + i\rho_{jy}$, $j = 1, 2$. Therefore, when acting on the vacuum $|\tilde{0}\rangle$, the operator $B_h^\dagger(\boldsymbol{\rho}_2) = \tilde{B}_h^\dagger(\mathbf{r}_h)$ gives a combination $(Z_2 + \frac{1}{\sqrt{2}}Z_1)/\sqrt{2}\ell_B = z_h/2\ell_B$. To eliminate the coordinate $\boldsymbol{\rho}_1$, we perform the shift $\boldsymbol{\rho}_1 \to \tilde{\boldsymbol{\rho}} = \boldsymbol{\rho}_1 + \frac{1}{\sqrt{2}}\boldsymbol{\rho}_2 = \frac{1}{\sqrt{2}}\mathbf{R}$ and obtain

$$\begin{aligned}\langle m_2 l_2|H_{\mathrm{eh}}|m_1 l_1\rangle = \int \frac{\mathrm{d}^2\rho_2}{2\cdot 2\pi\ell_B^2\sqrt{2^{l_1+l_2}l_1!l_2!}} &\times \exp\left(-\frac{\rho_2^2}{4\ell_B^2}\right)\int \mathrm{d}^2 r\,\phi_{0m_2}^{(e)*}(\mathbf{r})\frac{-2\sqrt{2}e^2}{\epsilon|\boldsymbol{\rho}_2 - \mathbf{r}|}\\ &\times \phi_{0m_1}^{(e)}(\mathbf{r})\int\frac{\mathrm{d}^2\tilde{\rho}}{2\pi\ell_B^2}\exp\left(-\frac{\tilde{\rho}^2}{2\ell_B^2}\right)\\ &\times\left(\frac{\tilde{Z}^*}{\sqrt{2}\ell_B} + \frac{Z_2^*}{2\ell_B}\right)^{l_2}\left(\frac{\tilde{Z}}{\sqrt{2}\ell_B} + \frac{Z_2}{2\ell_B}\right)^{l_1}\\ &\sim \delta_{l_1-m_1,l_2-m_2}\,.\end{aligned} \tag{22}$$

Integrating out the variable $\tilde{\boldsymbol{\rho}}$, we reduce the problem to an effective *two-particle* e–h problem in zero LLs (cf. [15]). The peculiarity of the situation is that the effective particles are characterized by *different* magnetic lengths. The matrix elements (22) can be presented in the form ($m_1 = m$, $m_2 = m + s$, $l_1 = l$, $l_2 = l + s$)

$$\langle m + s\, l + s|H_{\mathrm{eh}}|m\, l\rangle = (-2\sqrt{2})2^{-l-\frac{s}{2}}\sum_{k=0}^{l}\Big(C_l^k C_{l+s}^{k+s}\Big)^{\frac{1}{2}}U_{km}^{(\alpha=2)}(s)\,, \tag{23}$$

where C_n^m are binomial coefficients and the matrix elements of the Coulomb interparticle interactions in zero LLs have

Table 1
Singlet $X^-_{sn_en_h}$ and triplet $X^-_{tn_en_h}$ charged magnetoexcitons in Landau levels n_en_h

	M_z	Interaction energy (E_0)	Binding energy (E_0)
X^-_{t00}	-1^a	-1.04345	0.04345
X^-_{s01}	-3^b	-0.78056	0.20690
X^-_{t01}	-4^b	-0.75776	0.18410
X^-_{t10}	1^a	-1.08596	0.08596

[a] The only bound states in the given LLs n_en_h.
[b] The ground states among many other bound states with the same n_en_h.

been introduced:

$$\int d^2r_1 \int d^2r_2\, \phi^{(e)*}_{0m_2}(\mathbf{r}_1)\phi^{(h)*}_{0k_2}(\mathbf{r}_2/\sqrt{\alpha})\frac{e^2}{|\mathbf{r}_1-\mathbf{r}_2|}$$
$$\times\phi^{(h)}_{0k_1}(\mathbf{r}_2/\sqrt{\alpha})\phi^{(e)}_{0m_1}(\mathbf{r}_1)$$
$$=\delta_{k_1-m_1,k_2-m_2}U^{(\alpha)}_{\min(k_1,k_2),\min(m_1,m_2)}(|m_1-m_2|) \tag{24}$$

The matrix elements (24) can be found analytically for arbitrary α:

$$U^{(\alpha)}_{mn}(s)=E_0\frac{\alpha^{\frac{s}{2}}[m!(m+s)!n!(n+s)!]^{-\frac{1}{2}}}{(1+\alpha)^{s+\frac{1}{2}}\,2^{m+n+s}}$$
$$\times\sum_{k=0}^{m}\sum_{l=0}^{n}C^k_m C^l_n\frac{\alpha^l}{(1+\alpha)^{k+l}}[2(k+l+s)-1]!!$$
$$\times[2(m-k)-1]!!\,[2(n-l)-1]!!\,. \tag{25}$$

Eqs. (20), (23), and (25) determine the secular equation of the *infinite order* that should be solved to obtain the three-particle $2e$–h states in zero LLs. A truncation of the basis should naturally be performed. An important property of the developed basis (14) [and (18)] is that such a truncation *does not break* the translational invariance. On the contrary, a truncation of the basis (4), as performed in [9,10] (see also [7]), leads to spurious mixing of different k-states and violates the exact magneto-optical selection rule [14]—the conservation of the oscillator quantum number k.

The developed approach also provides an effective computational tool: First, we have been able to remove one degree of freedom in the three-particle problem, so that configurational space is substantially reduced (cf. [10]). As a result, with finite-size calculations it is even possible to reproduce with a reasonable accuracy the three-particle continuum—a neutral magnetoexciton plus a scattered electron [14]. Second, for *bound X^-* states lying outside the continua we have extremely rapid convergence within each LL. This is associated with the *exponential* decay of the off-diagonal matrix elements (23). Consider, e.g., the $k=0$ triplet $X^-_{tn_e=0n_h=0}$ state in zero LLs with $M_z=-1$. The asymptotic behavior of the relevant off-diagonal Coulomb e–h matrix elements is

$$\langle 2s+1\,2s|H_{\text{eh}}|10\rangle=-\frac{2\sqrt{2}}{2^s}U^{(\alpha=2)}_{01}(2s)$$
$$\approx-\sqrt{\frac{32}{27\pi}}\left(\frac{1}{9}\right)^sE_0\,,\; s\gg1\,. \tag{26}$$

Also, even the 1×1 matrix Hamiltonian in the basis (18) $\langle10|H_{\text{ee}}+H_{\text{eh}}|10\rangle=-1.0073E_0$ ensures the X^- binding: it gives a positive binding energy $0.0073E_0$; this is relative to the ground state energy $-E_0$ of the neutral $X_{n_e=0n_h=0}$ magnetoexciton in zero LLs. As a result, the X^-_{t00} binding energy can be calculated with virtually unlimited accuracy and equals $0.043452E_0$; this value is compatible with [7,10]. Not accounting for the Landau degeneracy in k, the X^-_{t00} state with $M_z=-1$ is the only low-lying bound X^- state in zero LLs: there are no other bound triplet or singlet states [7,10,14].

Similar considerations apply to the X^- states in higher LLs. Some of the results for the X^- ground states are presented in Table 1. There is only one bound X^- state in the first electron LL (the basis (14) includes the states with $n_1=1$, $n_2=0$, $n_h=0$ and $n_1=0$, $n_2=1$, $n_h=0$). This state is the triplet X^-_{t10} with $M_z=1$, whose binding energy is almost twice that of the X^-_{t00} state in zero LLs [14]. This resembles a stronger binding of the triplet D^- state (two electrons bound by a donor ion) in the first electron LL [16] and has the same physical origin. The X^-_{t10} binding energy is counted from the lowest possible unbound state in the same LL's, which is the neutral magnetoexciton $X_{n_e=0n_h=0}$ with the second electron in the scattering state in the $n_e=1$ LL. As calculations show, there are *many* bound X^- states in the next hole LL [$n_1=n_2=0$, $n_h=1$ in (14)]— both triplets X^-_{t01} and singlets X^-_{s01} (see Fig. 1). These are lying below the ground state of the neutral magnetoexciton $X_{n_e=0n_h=1}$; the latter has the energy $-0.5737E_0$. Due to this small binding energy of the neutral $X_{n_e=0n_h=1}$ magnetoexciton (comparatively to the $X_{n_e=0n_h=0}$ magnetoexciton), the triplet X^-_{t01} and singlet X^-_{s01} ground states have rather large binding energies (Table 1). In all LLs, there are also higher-lying bound three-particle $2e$–h states originating from the internal bound motion of 2D electrons in strong magnetic fields [14]. These states appear in the spectrum at relatively large positive values of the total M_z, that correspond to the hole being at large distances from the electrons (cf. with the similar states in the D^- problem [16]).

4. Conclusions

In conclusion, we have developed a formalism that allows one to preserve the exact symmetry—magnetic translations—when performing the Landau level expansion for charged electron–hole complexes in magnetic fields. This is achieved by using the Bogoliubov canonical transformation mixing the center-of-mass motions of the electron and

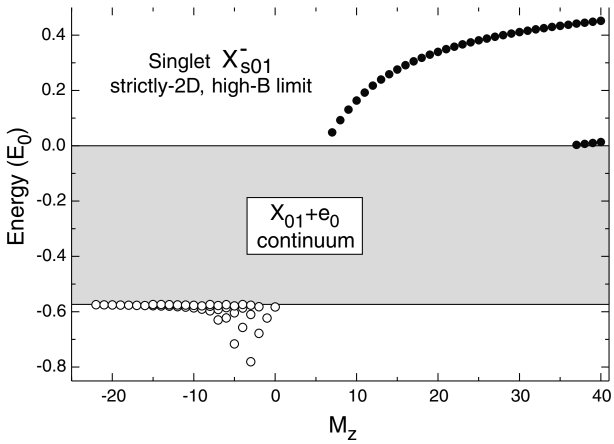

Fig. 1. Bound and scattering three-particle $2e$–h singlet $S_e = 0$ states in the $(n_e n_h) = (01)$ LLs. Note that only the $k = 0$ states are shown. The energy is given relative to the energy of free LLs $\frac{1}{2}\hbar\omega_{ce} + \frac{3}{2}\hbar\omega_{ch}$, in units of $E_0 = \sqrt{\pi/2}\, e^2/\epsilon l_B$. Open circles correspond to low-lying X^-_{s01} states. Filled circles correspond to excited states originating from bound internal motion of two electrons in 2D in high magnetic fields. The shaded area shows the continuum corresponding to the neutral magnetoexciton X_{01} plus a scattered electron in the $n_e = 0$ LL; the lower continuum edge lies at an energy $-0.5737E_0$.

hole subsystems. The effectiveness of the scheme has been demonstrated for high-accuracy and rapidly convergent calculations of two-dimensional charged excitons X^- in magnetic fields. This can be useful for studying the eigenspectra of charged excitons in quasi-two-dimensional quantum wells at strong and intermediate magnetic fields.

Acknowledgements

The author is grateful to H. Haug and A.Yu. Sivachenko for useful discussions. This work was supported by the Humboldt Foundation and the grants RBRF 97-2-17600 and “Nanostructures” 97-1072.

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