Quasitwo-dimensional electron–hole pair condensate in a strong magnetic field

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We succeeded in finding exact solutions for the grounds and several excited states in the model of a layered semiconductor or semimetal with interacting electrons (e) and holes (h) in a strong magnetic field. The system consists of two-dimensional planes with equal numbers of e and h, $N_{je} = N_{jh} = N_j$ (j being the number of the plane, and possibly $N_i \neq N_j$). The model takes exact account of all interactions, the interaction law V being arbitrary except that $V_{ee} = V_{hh} = -V_{eh}$. The transitions between Landau levels are ignored, as is permissible for strong fields.[1] Transitions between different planes are also ignored (the limit of strong anisotropy). Let us first consider the range of N_j and H for which the zero-order Landau level is occupied. We solve the many-particle Schrödinger equation with the Hamiltonian

$$\mathcal{H} = \sum \mathcal{H}_i + \sum \mathcal{H}_{ij},$$

$$\mathcal{H}_{ij} = \sum V_{ij}(x_1 x_2 \,|\, x_1' x_2') \Big[a^+_{ix_1} a^+_{jx_2} a_{jx_2'} a_{ix_1'} + (a \to b) - \sum_{\substack{k,\, l = i,\, j \\ k \neq l}} b^+_{kx_1} a^+_{lx_2} a_{lx_2'} b_{kx_1'} \Big], \tag{1}$$

where $\mathcal{H}_i$ is the interaction Hamiltonian for particles in layer i; a^+_{ix} and b^+_{ix} are the e and h creation operators in the Landau representation. The ground state of $\mathcal{H}_i$ is a Bose condensate of an ideal gas of two-dimensional excitons.[1,2] We shall use the algebraic method of equations of motion, which is more compact and suitable for similar problems that can be solved exactly.

We define the operators for creation of two-dimensional excitons in a strong field H, with momentum P, which enable us to take account of e and h pair correlations (cf. Ref. 3):

$$Q^+_{j\mathbf{P}} = \sum_x \varphi_{\mathbf{P}}(x)\, b^+_{jx+P_y/2} a^+_{jx-P_y/2}, \quad \varphi_{\mathbf{P}}(x) = N_0^{-1/2} \exp(iP_x x). \tag{2}$$

It is found that the states

$$|\Phi\rangle = \prod_j (Q^+_{j0})^{N_j} |0\rangle, \quad |\Phi_{i\mathbf{P}}\rangle = Q^+_{i\mathbf{P}} \prod_j (Q^+_{j0})^{N_j - \delta_{i,j}} |0\rangle, \tag{3}$$

corresponding to a Bose condensate of excitons and an exciton above the condensate, are exact eigenstates of $\mathcal{H}$:

$$\mathcal{H}|\Phi\rangle = \Big(\sum_j N_j E_0\Big)|\Phi\rangle, \quad \mathcal{H}|\Phi_{i\mathbf{P}}\rangle = \Big(\sum_j N_j E_0 + [E(\mathbf{P}) - E_0]\Big)|\Phi_{i\mathbf{P}}\rangle, \tag{4}$$

where $E(\mathbf{P}) = -1/S \sum U(q) \exp(iq\mathbf{P} - q^2/2)$ is the exciton dispersion relation at the zero-order Landau level,[1] $qU(q)$ the Fourier transform of the particle interaction potential in the layer. The proof of system (4) follows from the commutation relations $[\mathcal{H}_j, Q^+_{j0}] = E_0 Q^+_{j0}$, $[\mathcal{H}_{ij}, Q^+_{k0}] = 0$, which signify that zero momentum excitons formed in the layers do not interact with one another or with excitations above the condensate, a far from trivial result. The absence of interaction between excitons is a consequence of the exact compensation of all exchange and all direct (other than "intraexciton") interactions of e and h in a layer and of the compensation of all direct interactions between layers. The causes of the compensation of direct interactions are these: 1) all exciton multipole moments are zero; 2) the virtual transitions of excitons at a given Landau level with change of P are forbidden, and they therefore have no polarizational interactions.

Similarly, we consider equilibrium semimetals and semiconductors, in which the number of particles is found from the condition of minimum total energy (or is determined by the chemical potential fixed by another group of electrons). To get the number of particles, we construct a set of eigenstates corresponding to various numbers of particles and to the occupation of any number of Landau levels (these also correspond to a Bose condensate with P = 0) and separate the ground state. In the two-dimensional model, all Landau levels are either fully occupied or completely empty. Thus, if $\left| E_n + \sum_{m=0}^{n-1} E_{nm} \right| > 2n\omega_H - E_g(H)$ [where E_n is the binding energy of an exciton with P = 0 at Landau level n, E_{nm} the exchange interaction energy of two Landau levels, $E_g(H)$ the band overlap, $E_g(H) < 0$ for a semiconductor, and $2\omega_H = \omega_{He} + \omega_{Hh}$], the creation of e–h pairs at Landau level n is energetically favorable, and all Landau levels up to n are fully occupied.

The ground state in crossed electric (E) and magnetic fields, with a fully occupied Landau level, is again a Bose condensate, but with $\mathbf{P} = \mathbf{P}_0 = Mc\mathbf{E} \times \mathbf{H}/H^2$ ($M = m_e + m_h$), corresponding to the removal of the condensate with drift velocity $\mathbf{V}_0 = c\mathbf{E} \times \mathbf{H}/H^2$. When the Landau level is partly occupied, this state is exact but is not the ground state: the elongation of the exciton along the field E is more favorable. When the Landau level is fully occupied, the condensate with V_0 is evidently metastable with regard to interaction with a small number of impurities. This should cause, for example, an almost ideal exciton energy transfer in a system with nonequilibrium e and h (analogously to the case of the quantum Hall effect).

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