Exact solutions and Bogolyubov transformations of quasizero-dimensional electron–hole systems

A. B. Dzyubenko and Yu. E. Lozovik

P. N. Lebedev Physics Institute, Academy of Sciences of the USSR, Moscow

(Submitted December 9, 1982)

Fiz. Tverd. Tela (Leningrad) 25, 1519–1521 (May 1983)

PACS numbers: 71.35. + z, 71.10. + x

The ground state of electron–hole (e–h) quasizero-dimensional systems, i.e. two-dimensional (2D) in strong magnetic fields, is of considerable interest. It is shown in Ref. 1 that the ground state of such systems is an ideal (if transitions to the higher Landau levels are ignored) gas of noninteracting excitons.

However, this cannot be demonstrated by the usual diagrammatic methods at $T = 0$. The difficulty is the infinite (in the thermodynamic limit) Landau degeneracy. The proof given in Ref. 1 utilizes the temperature diagrammatic technique which allows for spontaneous symmetry breaking in a system and makes it possible to go to the limit $T \to 0$.

We shall consider an e–h system directly at $T = 0$. We shall use the Bogolyubov transformation method to obtain the exact wave function of the ground state which is in the form of a wave function of a Bose condensate of an ideal gas of excitons. To the best of our knowledge this is the first example of an exactly solvable two-dimensional problem for particles with the Coulomb interaction (see also Ref. 1).

We shall consider a nonequilibrium semiconductor in a magnetic field. The constant number of electrons N in the conduction band is equal to the number of holes in the valence band. The applied magnetic field is assumed to be strong: $r_H \ll a_B$, where $r_H^2 = c/eH$ is the magnetic length, $a_B = 1/me^2$ is the effective Bohr radius, and $\hbar = 1$ (see Ref. 1). If $\rho \equiv N/N_0 \leq 1$, then only the electron and hole Landau levels with $n = 0$ are occupied and we can ignore transitions to the remaining levels if $r_H \ll a_B$ ($N_0 = S/2\pi r_H^2$ is the degree of degeneracy of the Landau levels and S is the area of the system).

We shall adopt the Landau representation for the Hamiltonian of the system $\mathcal{H} = \mathcal{H}_{\rm int} - \mu\hat{N}$ [with the $\mathbf{A} = (0, Hx, 0)$ gauge; we shall also introduce identical chemical potentials e and h measured – in the nonequilibrium case – from the corresponding Landau levels; the interaction Hamiltonian $\mathcal{H}_{\rm int}$ is given by Eq. (3); $\hat{N} = \hat{N}_e + \hat{N}_h$ is the operator representing the total number of particles]. We shall transform $\mathcal{H}$ with the aid of a unitary operator

$$U = \exp\{\varphi(Q_0^+ - Q_0)\}, \tag{1}$$

where $Q_0^+ = \sum b_{p_y}^+ a_{p_y}^+$ is, as can be shown, the operator of creation of a quasizero-dimensional exciton with a momentum $P = 0$; $a_{p_y}^+$ and $b_{p_y}^+$ are the creation operators of electrons (e) and holes (h) with the quantum number p_y at the level $n = 0$. The transformed Hamiltonian $\tilde{\mathcal{H}} = U\mathcal{H}U^+$ has the form $\tilde{\mathcal{H}} = W + \tilde{\mathcal{H}}_0 + \tilde{\mathcal{H}}_{\rm int}$, where $W(\varphi) = -(2\mu - E_0)v^2N_0$ is a numerical function, $\tilde{\mathcal{H}}_0$ is the one-particle part of the Hamiltonian

$$\tilde{\mathcal{H}}_0 = uv\,[2\mu - E_0] \sum (b_{p_y}^+ a_{p_y}^+ + a_{p_y} b_{p_y}) + [v^2(2\mu - E_0) - \mu] \sum (a_{p_y}^+ a_{p_y} + b_{p_y}^+ b_{p_y}), \tag{2}$$

$\tilde{\mathcal{H}}_{\rm int}$ is the interaction Hamiltonian

$$\tilde{\mathcal{H}}_{\rm int} = \mathcal{H}_{\rm int} = \frac{1}{2} \sum V(12\,|\,1'2')\,[a_1^+ a_2^+ a_{2'} a_{1'} + (a \to b) - 2b_{1'}^+ a_2^+ a_{2'} b_1], \tag{3}$$

$u = \cos\varphi$, $v = \sin\varphi$, and E_0 is the binding energy of a quasizero-dimensional exciton. The above results apply to a fairly wide range of interactions V [in the Coulomb case we have $E_0 = -(\pi/2)^{1/2}(e^2/r_H)$ – Ref. 1]. A remarkable property of Eq. (3) is that $\tilde{\mathcal{H}}_{\rm int}$ has no terms that describe pair (one or more) creation from vacuum and which make it unstable (compare with Ref. 2, where the 3D case is considered). We shall compensate the off-diagonal terms in $\tilde{\mathcal{H}}_0$ by applying to the chemical potential μ the condition

$$2\mu = E_0. \tag{4}$$

We can easily show that if Eq. (4) is obeyed, a BCS-type state $|\Phi\rangle \equiv U^+|0\rangle$ represents an eigenvalue of the original Hamiltonian $\mathcal{H}$ ($|0\rangle$ represents vacuum for a_{p_y} and b_{p_y}):

$$\mathcal{H}\,|\Phi\rangle = W(\varphi)\,|\Phi\rangle = 0. \tag{5}$$

The condition for the normalization of the number of particles in the $|\Phi\rangle$ state gives the parameter φ: $v^2 = \sin^2\varphi = \rho$. We shall now subject Eq. (5) to the operator P_N representing projection on states with N electrons and N holes, and we shall allow for the fact that it commutes with $\mathcal{H}$ ($\mathcal{H}$ conserves the number of particles). We then obtain the exact eigenstate of $\mathcal{H}_{\rm int}$ in the form of a wave function of a Bose condensate of ideal excitons [an unimportant factor is omitted from Eq. (7)]:

$$\mathcal{H}_{\rm int}\,|\Phi_N\rangle = NE_0\,|\Phi_N\rangle, \tag{6}$$

$$|\Phi_N\rangle \equiv P_N\,|\Phi\rangle = (Q_0^+)^N\,|0\rangle \tag{7}$$

[Eq. (6) can easily be demonstrated directly].

We can see that the ground state of such a system is less than the energy of a noninteracting e–h system and the difference is equal to the binding energy of N excitons which amounts to NE_0. The condition (4) is equivalent to $\mu = 0$ for an ideal Bose gas below the Bose condensation point (if the chemical potential is measured from the exciton binding energy E_0). In accordance with Ref. 1, it follows from our wave function that the energy of the system is not affected when the area of the system is altered but the number of particles remains constant. An allowance for transitions to the higher Landau levels removes this property and makes a Bose exciton gas slightly nonideal (the interaction depends on the field H).

 0038-5654/83/05 0874-02 \$03.90

The authors are grateful to L. V. Keldysh, D. A. Kirzhnits, and participants of a seminar headed by V. L. Ginzburg for valuable advice.

[1]I. V. Lerner and Yu. E. Lozovik, Pis'ma Zh. Eksp. Teor. Fiz. 27, 497 (1978) [JETP Lett. 27, 467 (1978)]; Zh. Eksp. Teor. Fiz. 80, 1488 (1981); 82, 1188 (1982) [Sov. Phys. JETP 53, 763 (1981); 55, 691 (1982)].

[2]L. V. Keldysh and A. N. Kozlov, Zh. Eksp. Teor. Fiz. 54, 978 (1968) [Sov. Phys. JETP 27, 521 (1968)].

Translated by A. Tybulewicz

 0038-5654/83/05 0875-02 $03.90