CHAPTER

1

SURVEY OF PARTICLES AND FORCES

If in some cataclysm, all of scientific knowledge were to be destroyed, and only one sentence passed on to the next generations of creatures, what statement would contain the most information in the fewest words? I believe it is the *atomic hypothesis* (or the atomic *fact*, or whatever you wish to call it) that *all things are made of atoms—little particles that move around in perpetual motion, attracting each other when they are a little distance apart, but repelling upon being squeezed into one another.* In that one sentence, you will see, there is an enormous amount of information about the world, if just a little imagination and thinking are applied.

Richard P. Feynman

Matter is made of atoms. The properties of atoms are quite remarkable. Consider an ordinary rock. Try pulling the atoms in a rock apart or squeezing them together. It is not easy to do so! The atoms in the rock are remarkably stable. The discovery of atoms and the measurement of their properties have paved the way for our present understanding of the universe. The idea of matter being composed of atoms is the single most important concept in all of science. The atomic composition of matter explains such apparently diverse phenomena as why the sky looks blue, why a rock feels hard, why a rose smells fragrantly, why a violin sounds mellow, and why a lime tastes sour. Our story of modern physics begins by tracing the important ideas and experiments leading to the discovery of atoms.

1-1 DISCOVERY OF ATOMS

About 2400 years ago, the Greek philosopher Anaxagoras invented the idea that matter was composed of tiny invisible seeds, or spermata. This concept was expanded a few years later by Democritus, who called the indivisible particles of matter *atoms*. The atomic hypothesis had its renaissance in the nineteenth century as scientists made the famous classification of the elements in the form of the periodic table. The idea of explaining the properties of a complex object with elementary building blocks has survived from the ancient Greeks into modern science. We *know* that matter is composed of atoms because we have developed the experimental techniques needed to *test* the atomic hypothesis. The ancient Greeks did not have the necessary experimental tools; this is why there was no advance in the understanding of atoms for more than 2000 years!

Atomic Mass Numbers

The experimental foundation of the atomic theory is the *law of definite proportions*: Whenever a given compound is formed from two elements, the ratio of the combining masses of the elements is observed to be a constant. This result holds for every compound although the mass ratio is different for each compound. If a compound is made up of more than two elements, then the ratio of masses of any two elements is constant.

In 1807, John Dalton postulated that atoms of each element had a unique mass. Dalton's atomic theory contained a simple prediction for the case where the same two elements combine to form two different compounds: For a given mass of one of the combining elements, the masses of the other element needed to make the two compounds must be in the ratio of two small integers. The Dalton atomic theory was quickly proven to be correct by experiment. (For example, 16 g of oxygen combines with 12 g of carbon to form carbon monoxide and 32 g of oxygen combines with 12 g of carbon to form carbon dioxide. The ratio of oxygen masses needed to make the two compounds is 2/1.) This result is known as the *law of multiple proportions*. According to the theory of Dalton, each element was assigned an integer *atomic mass number* (A). Scientists of the early nineteenth century faced the formidable problem of determining both the atomic masses of the elements and the chemical formulas of compounds.

A great leap forward in the understanding of the structure of matter was made in 1811 by Amedeo Avogadro. Avogadro correctly hypothesized that the particles of a gas were small in size compared to the distance between the particles. Avogadro determined that the particles of the gas were often made up of more than one atom bound together into *molecules* and that at a fixed temperature and pressure, equal volumes of a gas contained equal numbers of molecules. This important result, which will be discussed in much more detail in Chapter 2, is the basis of the *ideal gas law*.

The *molecular mass number* is defined to be the sum of the atomic mass numbers of the atoms that make up the molecule. Relative molecular mass numbers of compounds were determined by measuring the masses of equal volumes of gases at fixed temperature and pressure. Together with the assumption that the simplest molecules contained only one atom of certain elements, the discovery of Avogadro provided a systematic method for measurement of the atomic mass numbers.

The Periodic Table

In 1869, Dmitri Mendeleev made the first classification of the elements according to their chemical properties and their atomic mass numbers. The elements were ordered with increasing atomic mass number and placed in several columns according to their chemical properties. Starting with hydrogen, an integer serial number was assigned sequentially to each element. This serial number is called the *atomic number* (Z). For hydrogen $Z = 1$, for helium $Z = 2$, and so on. In his periodic table, Mendeleev discovered some gaps that allowed him to correctly predict the existence of undiscovered elements, the ultimate goal of a theoretician! The missing elements were soon discovered. All was fine with the periodic table until William Ramsay and Lord Rayleigh discovered the element argon in 1894.

Argon had no place in the theoretical classification of the elements; such a discovery is the ultimate goal of an experimentalist! The periodic table was modified by adding a whole extra column to accommodate argon and other inert gases that were soon discovered. All the great advancements in science have been made through such interplay between theory and experiment. The modern periodic table of the elements is shown in Figure 1-1.

Avogadro's Number

Once the atomic mass numbers of the elements were known, scientists had a very powerful atomic relationship: There are equal numbers of atoms in A grams of any element, where A is the atomic mass number of the element. For example, 1 g of hydrogen, 12 g of carbon, and 238 g of uranium all contain the same number of atoms (see Figure 1-1). The number of atoms in A grams of any element is called *Avogadro's number* (N_A). The quantity of matter comprising Avogadro's number of atoms is called one *mole*. The next great experimental challenge was to determine the value of Avogadro's number. *Just how many atoms are there in one gram of hydrogen?*

Measuring the Size of an Atom

Consider the measurement of the size of an object using light as a probe. Suppose that the object to be measured is the width of a narrow slit, as illustrated in Figure 1-2. Rays of light are allowed to pass through the slit, and the intensity of the light is measured at a large distance from the slit. The image of the narrow slit is not infinitely sharp because the rays of light bend or *diffract* on passing through the slit. Diffraction is a fundamental property of waves. The location of the maxima and minima of the diffraction pattern may be deduced by tracing rays of light through the slit. Destructive interference occurs when rays have path lengths that differ by an amount (ΔL) equal to one-half of the wavelength of the light rays (λ_{light}):

$$\Delta L = \frac{\lambda_{\text{light}}}{2}. \tag{1.1}$$

If the width of the slit is d, then the path length difference is related to the angle at which the intensity is a minimum (θ_{min}) by

$$\Delta L = \frac{d}{2}\sin\theta_{\text{min}}. \tag{1.2}$$

Combining these results gives

$$\lambda_{\text{light}} = d\sin\theta_{\text{min}}. \tag{1.3}$$

Measurement of θ_{min} determines the size d of the slit. The sharpness of the intensity pattern, which is governed by diffraction, is directly proportional to the wavelength of the light.

For $\lambda_{\text{light}} = d$, $\theta_{\text{min}} = \pi/2$ and destructive interference is not measurable. We cannot measure the size of the slit using light that has a wavelength larger than the size of the slit. For this case, all we can experimentally determine is an upper limit on the slit size, $d < \lambda_{\text{light}}$.

As a result of diffraction, measurement of the size of an object is limited by the wavelength of the light used in the measurement. Two points separated by a distance d can be resolved only if the wavelength of the light does not exceed d. A consequence of this is that a single atom cannot be resolved with an ordinary microscope. This has nothing to do with the quality of the microscope, but rather with the fundamental limit imposed by diffraction. The wavelength of light, defined by the sensitivity of the eye, is in the range

$$400\ \text{nm} < \lambda_{\text{light}} < 700\ \text{nm}. \tag{1.4}$$

One nanometer (nm) is equal to 10^{-9} meters. The diameter of an atom (d_{atom}) is much smaller than the wavelength of light:

$$d_{\text{atom}} << \lambda_{\text{light}}. \tag{1.5}$$

The microscope *was* used, however, to make the first determination of the size of an atom! This grew out of the discovery in 1828 by Robert Brown that small particles suspended in a liquid have a small but measurable random motion. This *Brownian* motion is caused by molecules of the liquid colliding randomly with the suspended particles. The average displacement as a function of time depends on the rate at which molecules strike the suspended particle. The rate at which the molecules strike the suspended particle depends on the number of molecules in the liquid.

In 1905, Albert Einstein published a famous paper on the molecular theory of heat. From his molecular theory, Einstein deduced a formula for the time (t) dependence of the average displacement (R) of a sphere of known radius (r_0),

$$R = C\sqrt{\frac{t}{N_A r_0}}, \tag{1.6}$$

where C is a constant for a given liquid at a fixed temperature. (The meaning of temperature is an important concept that is the subject of Chapter 2.)

Periodic Table of the Elements

atomic number (Z) atomic mass (A)
name
symbol
density (10^3 kg/m^3)

1 1.01 hydrogen H 0.0708																	2 4.00 helium He 0.125
3 6.94 lithium Li 0.542	4 9.01 beryllium Be 1.82											5 10.8 boron B 2.47	6 12.0 carbon C 3.52	7 14.0 nitrogen N 0.808	8 16.0 oxygen O 1.14	9 19.0 fluorine F 1.11	10 20.2 neon Ne 1.21
11 23.0 sodium Na 1.01	12 24.3 magnesium Mg 1.74											13 27.0 aluminum Al 2.70	14 28.1 silicon Si 2.33	15 31.0 phosphorous P 1.82	16 32.1 sulfur S 2.07	17 35.5 chlorine Cl 1.56	18 40.0 argon Ar 1.40
19 39.1 potassium K 0.910	20 40.1 calcium Ca 1.53	21 45.0 scandium Sc 2.99	22 47.9 titanium Ti 4.51	23 50.9 vanadium V 6.09	24 52.0 chromium Cr 7.19	25 54.9 manganese Mn 7.47	26 55.9 iron Fe 7.87	27 58.9 cobalt Co 8.9	28 58.7 nickel Ni 8.91	29 63.6 copper Cu 8.93	30 65.4 zinc Zn 7.13	31 69.7 gallium Ga 5.91	32 72.6 germanium Ge 5.32	33 74.9 arsenic As 5.77	34 79.0 selenium Se 4.81	35 79.9 bromine Br 3.12	36 83.8 krypton Kr 3.09
37 85.5 rubidium Rb 1.63	38 87.6 strontium Sr 2.58	39 88.9 yttrium Y 4.48	40 91.2 zirconium Zr 6.51	41 92.9 niobium Nb 8.58	42 95.9 molybdenum Mo 10.2	43 98 technetium Tc 11.5	44 101 ruthenium Ru 12.4	45 103 rhodium Rh 12.4	46 106 palladium Pd 12.0	47 108 silver Ag 10.5	48 112 cadmium Cd 8.65	49 115 indium In 7.29	50 119 tin Sn 5.76	51 122 antimony Sb 6.69	52 128 tellurium Te 6.25	53 127 iodine I 4.95	54 131 xenon Xe 3.52
55 133 cesium Cs 2.00	56 137 barium Ba* 3.59	71 175 lutetium Lu 9.84	72 178 hafnium Hf 13.2	73 181 tantalum Ta 16.7	74 184 tungsten W 19.3	75 186 rhenium Re 21.0	76 190 osmium Os 22.6	77 192 iridium Ir 22.6	78 195 platinum Pt 21.5	79 197 gold Au 19.3	80 201 mercury Hg 14.3	81 204 thallium Tl 11.9	82 207 lead Pb 11.3	83 209 bismuth Bi 9.80	84 209 polonium Po 9.31	85 210 astatine At	86 222 radon Rn
87 223 francium Fr	88 226 radium Ra†	103 260 lawrencium Lr	104 261 unnilquadium Unq	105 262 unnilpentium Unp	106 263 unnilhexium Unh	107 262 unnilseptium Uns	108 265 unniloctium Uno	109 266 unnilennium Une									

* Lanthanide series	57 139 lanthanum La 6.17	58 140 cerium Ce 6.77	59 141 praseodymium Pr 6.78	60 144 neodymium Nd 7.00	61 145 promethium Pm	62 150 samarium Sm 7.54	63 152 europium Eu 5.24	64 157 gadolinium Gd 7.89	65 159 terbium Tb 8.27	66 163 dysprosium Dy 8.53	67 165 holmium Ho 8.80	68 167 erbium Er 9.04	69 169 thulium Tm 9.32	70 173 ytterbium Yb 6.97
† Actinide series	89 227 actinium Ac 10.1	90 232 thorium Th 11.7	91 231 protactinium Pa 15.4	92 238 uranium U 19.1	93 237 neptunium Np 20.5	94 244 plutonium Pu 19.8	95 243 americium Am 11.9	96 247 curium Cm	97 247 berkelium Bk	98 251 californium Cf	99 252 einsteinium Es	100 257 fermium Fm	101 258 mendelevium Md	102 259 nobelium No

FIGURE 1-1 Periodic table of the elements.
Elements in the same column have similar properties. The lanthanide and actinide series elements have properties similar to La and Ac. Each element has a unique atomic number (Z). For a given element, the atomic mass number (A) is not unique. The value of A in the table corresponds to the weighted average of the abundance on earth. If the element is not stable, the value of A corresponds to the isotope with the longest lifetime.

In 1908, Jean-Baptiste Perrin developed a technique for manufacturing small resin spheres of uniform size ($r_0 \approx 10^{-6}$ m = 1 μm). Perrin accurately measured the Brownian motion of these spheres and used Einstein's result to determine N_A. He checked the validity of Einstein's formula (1.6) by varying the size of the sphere and the composition of the liquid in which the spheres were suspended. Measurements made by Perrin on a single sphere are shown in Figure 1-3. Figure 1-3*a* shows the (x,y) position of the sphere at 30-second intervals. Figure 1-3*b* shows the net displacement versus the square root of time. From the measurement the displacement (1.6) of many spheres, Perrin found Avogadro's number to be about 6×10^{23}.

Perrin (with help from Einstein!) had determined the number of atoms in A grams of any element.

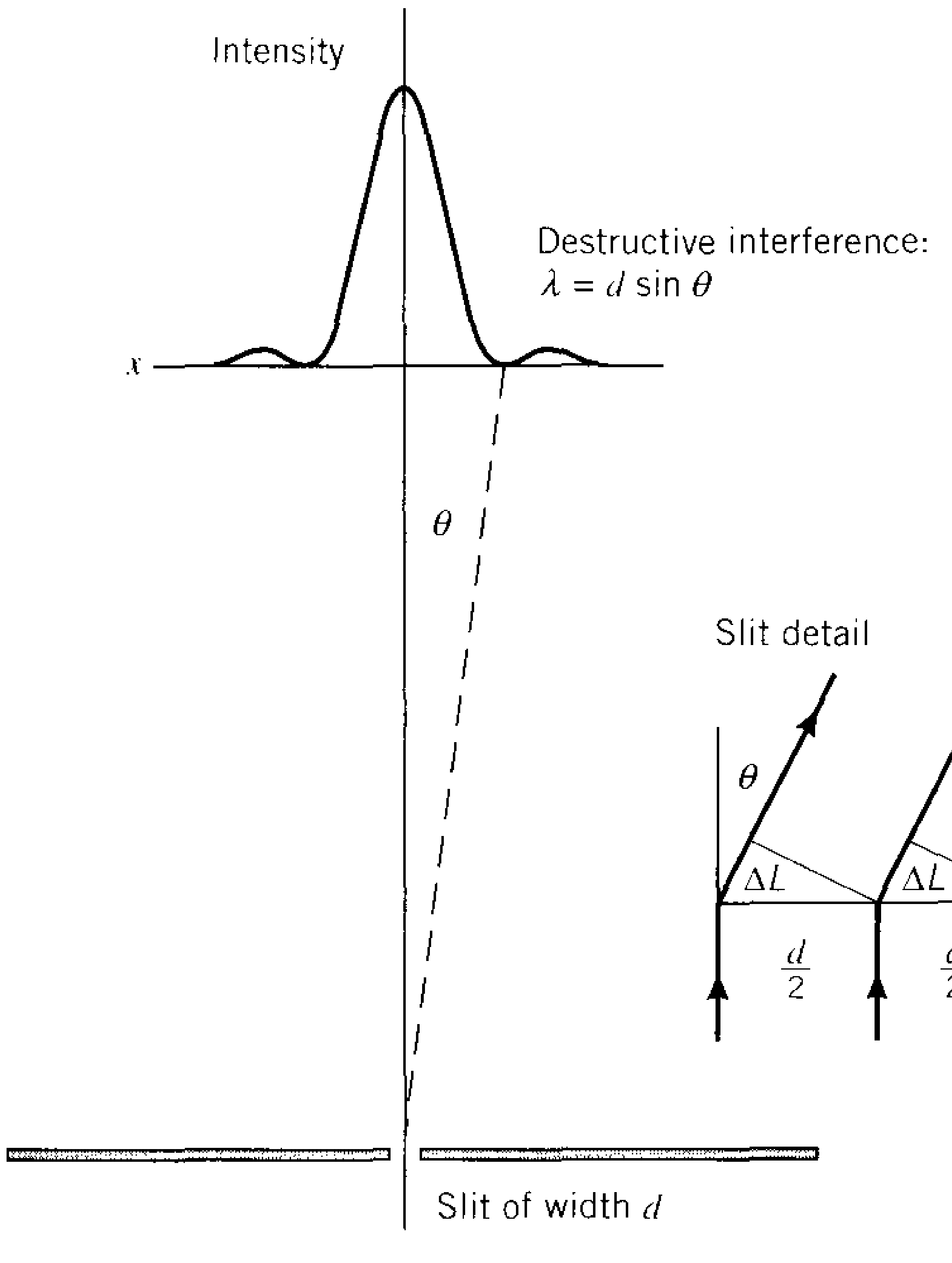


FIGURE 1-2 Diffraction of light.
An image of a narrow slit is made by measuring the intensity distribution of light that reaches a detector placed a large distance from the slit. The image is not perfectly sharp because of the diffraction of the light waves.

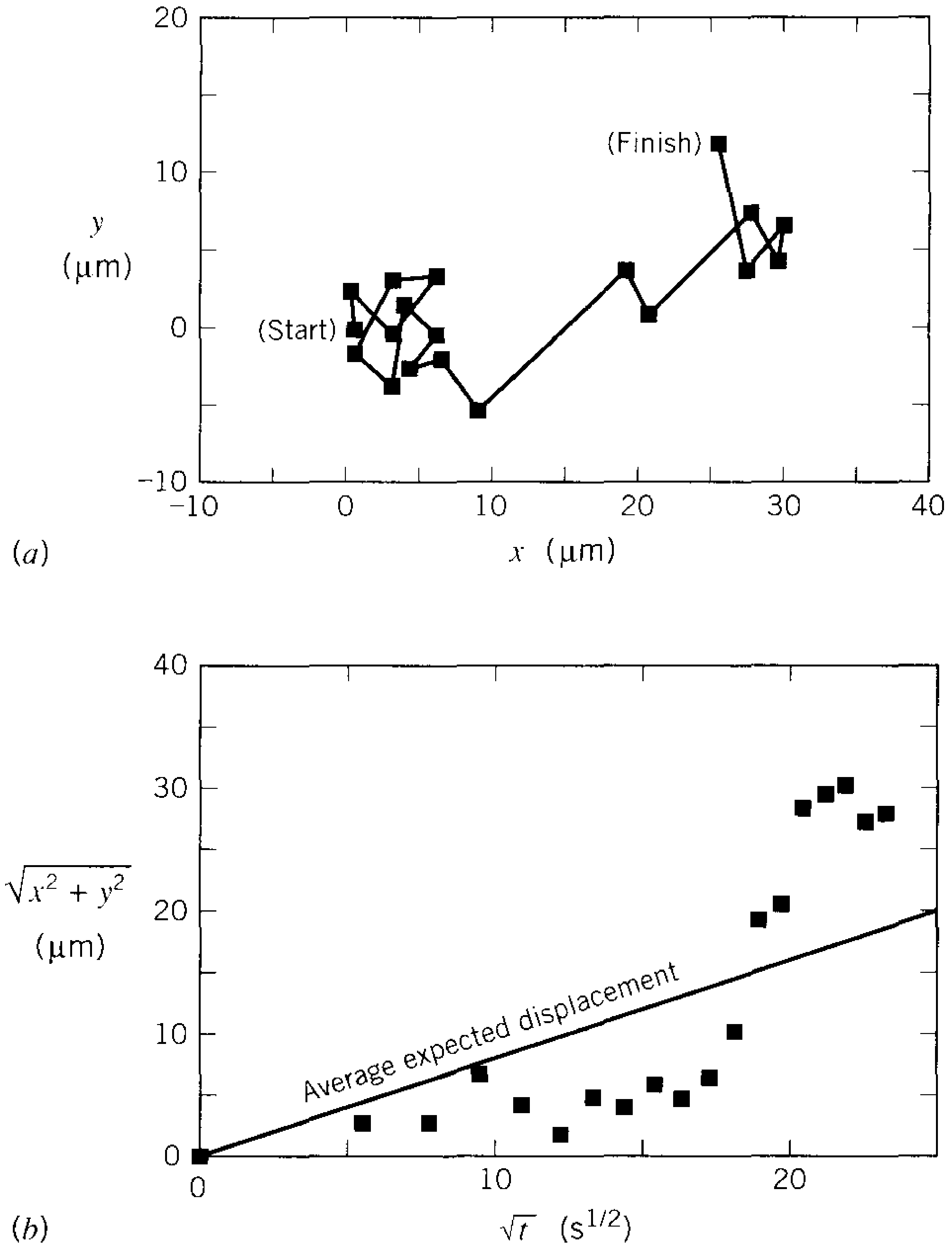


FIGURE 1-3 Data of Perrin on Brownian motion.
A tiny sphere of resin (r_0 = 0.5 μm) is suspended in a liquid. The sphere is observed to move when viewed under a microscope because of collisions with molecules of the liquid. (*a*) The measured position (x,y) of a single sphere is plotted at 30-second intervals. (*b*) The net displacement for the sphere is plotted versus the square root of time. Einstein predicted that the average net displacement is proportional to the square root of time. The constant of proportionality contains $N_A^{-1/2}$. The deviations from the average for any one sphere may be quite large, as indicated. The results of many measurements by Perrin proved that Einstein's prediction was correct and provided an accurate determination of Avogadro's number. Data are taken from J.-B. Perrin, *Atoms*, D. Van Nostrand (1923).

To be precise, Avogadro's number has been defined to be the number of atoms in 12 g of carbon (A = 12). Avogadro's number may be accurately determined by measuring the mass of a single carbon atom and then dividing 12 g by the atomic mass. The result is

$$N_A = 6.02 \times 10^{23}. \qquad (1.7)$$

(Precise values of physical constants may be found in Appendix A.) The atomic mass numbers of all the other elements are defined so that A grams of every element contains exactly N_A atoms. If we assume that the mass of a molecule is equal to the sum of the masses of the atoms that make up the molecule, then for any compound there are N_A molecules in M grams, where M is the sum of the atomic mass numbers of the atoms making up one molecule.

> Avogadro's number is
>
> $$N_A = 6.02 \times 10^{23}.$$
>
> For any element there are N_A atoms in A grams, where A is the atomic mass number.

EXAMPLE 1-1

Use Avogadro's number to calculate the mass of the hydrogen atom.

SOLUTION:

Since $A = 1$, 1 g of hydrogen contains N_A atoms. The mass of a single atom (m_H) is

$$m_H = \frac{A\left(10^{-3}\text{ kg}\right)}{N_A} = \frac{(1)\left(10^{-3}\text{ kg}\right)}{6.02 \times 10^{23}} \approx 1.7 \times 10^{-27}\text{ kg}. \blacksquare$$

Matter in a condensed state (solid or liquid) is incompressable. Combining Avogadro's number with the density of a solid or liquid gives us the approximate size of an atom.

EXAMPLE 1-2

The density of liquid hydrogen is about 71 kg/m^3. Use Avogadro's number to estimate the size of the hydrogen atom.

SOLUTION:

There are N_A atoms in 1 g of hydrogen. In the liquid state, the atoms are packed closely together. The volume (V) occupied by 1 g of hydrogen is

$$V = \frac{1.0 \times 10^{-3}\text{ kg}}{\rho}.$$

If we divide this quantity by Avogadro's number, we get the volume occupied by one atom (V_{atom}):

$$\begin{aligned} V_{\text{atom}} &= \frac{V}{N_A} \approx \frac{1.0 \times 10^{-3}\text{ kg}}{\rho N_A} \\ &\approx \frac{1.0 \times 10^{-3}\text{ kg}}{\left(71\text{ kg/m}^3\right)\left(6.0 \times 10^{23}\right)} \approx 2.3 \times 10^{-29}\text{ m}^3. \end{aligned}$$

This is the volume taken up by a single atom in liquid hydrogen. If we approximate the liquid as a collection of closely packed spheres, then the diameter of the atom (d_{atom}) is approximately

$$\begin{aligned} d_{\text{atom}} &\approx V_{\text{atom}}^{1/3} \approx \left(2.3 \times 10^{-29}\text{ m}^3\right)^{1/3} \\ &\approx 3 \times 10^{-10}\text{ m} = 0.3\text{ nm} \end{aligned} \quad \blacksquare$$

EXAMPLE 1-3

Estimate the size of a uranium atom. The density of uranium is 1.9×10^4 kg/m^3.

SOLUTION:

The atomic mass number of uranium is 238. Following the previous example, we have

$$\begin{aligned} V_{\text{atom}} &\approx \frac{(238)\left(10^{-3}\text{ kg}\right)}{\left(1.9 \times 10^4\text{ kg/m}^3\right)\left(6.0 \times 10^{23}\right)} \\ &\approx 2.1 \times 10^{-29}\text{ m}^3, \end{aligned}$$

and

$$\begin{aligned} d_{\text{atom}} &\approx V_{\text{atom}}^{1/3} \approx \left(2.1 \times 10^{-29}\text{ m}^3\right)^{1/3} \\ &\approx 3 \times 10^{-10}\text{ m} = 0.3\text{ nm}. \end{aligned} \quad \blacksquare$$

The sizes of the atoms in solids and liquids calculated in the fashion of Examples 1-2 and 1-3 are shown in Figure 1-4. These data show that atoms of the different elements are all of the same order of magnitude in size, even though they vary in mass by more than two orders of magnitude. The diameter of an atom (d_{atom}) is approximately

$$d_{\text{atom}} \approx 3 \times 10^{-10}\text{ m} = 0.3\text{ nm}. \tag{1.8}$$

Feynman has given us a clever way to visualize the size of an atom:

> *if an apple is magnified to the size of the earth, then the atoms in the apple are approximately the size of the original apple.*

> The diameter of an atom (d_{atom}) is approximately equal to
>
> $$d_{\text{atom}} \approx 3 \times 10^{-10}\text{ m} = 0.3\text{ nm}.$$

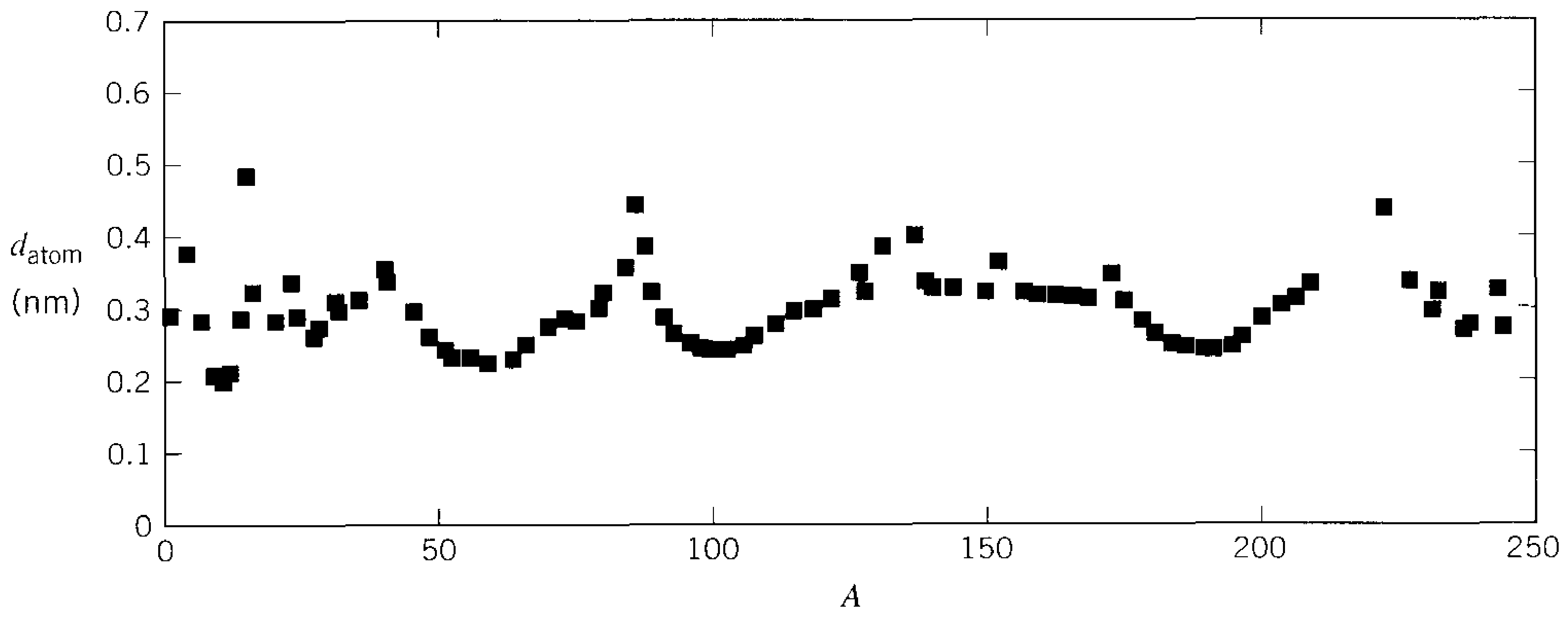


FIGURE 1-4 Approximate diameters of atoms in condensed matter as a function of the atomic mass number.

1-2 CLASSICAL ELECTROMAGNETISM

The cornerstone for the description of the electric force is Coulomb's law, which gives us an expression for the force (**F**) between two charges (q_1 and q_2) *at rest*:

$$\mathbf{F} = \frac{kq_1q_2}{r^2}\mathbf{i}_r. \tag{1.9}$$

The force is directed along the axis ($\mathbf{i}_r$) of the two charges. The electric force constant (k) together with the value of the electric charges specifies the strength of the interaction. Coulomb's law holds only for charges at rest. If the charges are moving, the expression for the force is much more complicated! Fortunately, a very ingenious method was invented that greatly simplifies the description of the force through the concept of electric and magnetic fields. Once the fields are known in some region of space, they may be used to calculate the force acting on a charge q moving with a velocity **v**:

$$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B}). \tag{1.10}$$

This expression is called the *Lorentz force law*. An electric field of 1 volt per meter produces the same magnitude of force on a charge as a magnetic field of 1 tesla when the charge has a speed of 1 meter per second: 1 V/m = 1 T·m/s.

The Superposition Principle

Coulomb's law gives us the electromagnetic force between two charges at rest. *How do we find the forces between three charges at rest?* The electromagnetic force has the very remarkable property that the force between two charges does not depend on the presence of the third charge! The resultant force on one charge is the vector sum of the forces due to the other two charges. This important experimental result is known as the *superposition principle*. The superposition principle can also be stated in terms of electric and magnetic fields. The net electric and magnetic fields due to many charges ($\mathbf{E}_{\text{net}}$ and $\mathbf{B}_{\text{net}}$) are given by the vector sum of the fields ($\mathbf{E}_1$, $\mathbf{E}_2$, $\mathbf{E}_3$ . . . and $\mathbf{B}_1$, $\mathbf{B}_2$, $\mathbf{B}_3$ . . .) due to the individual charges:

$$\mathbf{E}_{\text{net}} = \mathbf{E}_1 + \mathbf{E}_2 + \mathbf{E}_3 + \ldots, \tag{1.11}$$

and

$$\mathbf{B}_{\text{net}} = \mathbf{B}_1 + \mathbf{B}_2 + \mathbf{B}_3 + \ldots \tag{1.12}$$

We may use the superposition principle and the Lorentz force law (1.10) to calculate the total force on a charge q moving with velocity **v**:

$$\mathbf{F} = q\left(\mathbf{E}_{\text{net}} + \mathbf{v} \times \mathbf{B}_{\text{net}}\right). \tag{1.13}$$

Maxwell's Equations

The determination of the expressions for the electric and magnetic fields of moving charges, by Faraday, Ampère, and others, was a great scientific accomplishment. The crowning achievement was due to James Clerk Maxwell, who provided a unified set of four equations relating the charges and fields. Maxwell's equations together with the Lorentz force law summarize all that was known about the electromagnetic force at the end of the nineteenth century. Table 1-1 gives a brief summary of Maxwell's equations in integral form. These equations are also often written as differential equations (see Appendix B).

TABLE 1-1
SUMMARY OF MAXWELL'S EQUATIONS.

Gauss's Law	Gauss's Law for Magnetic Fields	Faraday's Law	Ampère's Law
$\oiint d\mathbf{a} \cdot \mathbf{E} = 4\pi k q_{\text{tot}}$	$\oiint d\mathbf{a} \cdot \mathbf{B} = 0$	$\oint d\mathbf{l} \cdot \mathbf{E} = -\frac{\partial}{\partial t} \oiint d\mathbf{a} \cdot \mathbf{B}$	$\oint d\mathbf{l} \cdot \mathbf{B} = \frac{4\pi k I}{c^2} + \frac{1}{c^2} \frac{\partial}{\partial t} \oiint d\mathbf{a} \cdot \mathbf{E}$
Flux of electric field through any closed surface is proportional to the electric charge contained inside the volume enclosed by that surface	There are no magnetic charges (*monopoles*)	Line-integral of the electric field around a closed loop is equal to the negative of the time-rate of change of the magnetic flux through the surface enclosed by the loop	Line-integral of the magnetic field around a closed loop is the sum of two terms, one proportional to the current through that loop and the second proportional to the time-rate of change of the electric flux through the loop

The Wave Equation

All of classical electrodynamics is contained in the four Maxwell equations plus the Lorentz force law. Unlike Newton's Second Law ($\mathbf{F} = m\mathbf{a}$), Maxwell's equations are relativistically correct. They have the same form even when the speeds of the charges are not small compared to the speed of light. It is not hard to write down Maxwell's equations or to find them on a tee-shirt. To appreciate the implication of Maxwell's equations, that an accelerated charge radiates electromagnetic waves, requires a much more sophisticated level of understanding.

An electromagnetic wave consists of oscillating electric and magnetic fields, which are perpendicular to each other and to the direction of travel of the wave (see Figure 1-5). The wave is able to propagate in vacuum because the changing electric field creates a magnetic field (Ampère's law) and the changing magnetic field creates an electric field (Faraday's law). The wave equation relates the second-order spatial and time derivatives of each component of the electric and magnetic fields. The wave equation may be derived from the Maxwell equations. The result (see Appendix B) is

$$\frac{\partial^2 F}{\partial x^2} + \frac{\partial^2 F}{\partial y^2} + \frac{\partial^2 F}{\partial z^2} = \frac{1}{c^2} \frac{\partial^2 F}{\partial t^2}. \tag{1.14}$$

An equation of the same form is satisfied for every component of both **E** and **B**, so that the $F(x,y,z,t)$ in the wave equation (1.14) can represent E_x, E_y, E_z, B_x, B_y or B_z. The speed of propagation of the wave is c, the speed of light in vacuum. It was the appearance of c in Ampère's law that led Maxwell to the brilliant deduction that light is an electromagnetic wave.

Consider an electromagnetic wave propagating in the z direction. The wave equation (1.14) becomes

$$\frac{\partial^2 F}{\partial t^2} = c^2 \frac{\partial^2 F}{\partial z^2}. \tag{1.15}$$

The solution is a function of the form $F(z-ct)$ or $F(z+ct)$ as can be seen by direct differentiation. Taking the electric field to be in the x direction, we may write the solution as

$$\mathbf{E} = F(z-ct)\mathbf{i}_x. \tag{1.16}$$

From Faraday's law the magnetic field must be in the y direction and have a strength

$$|\mathbf{B}| = \frac{|\mathbf{E}|}{c} \hat{\imath} = \frac{1}{c} F(z-ct)\mathbf{i}_y. \tag{1.17}$$

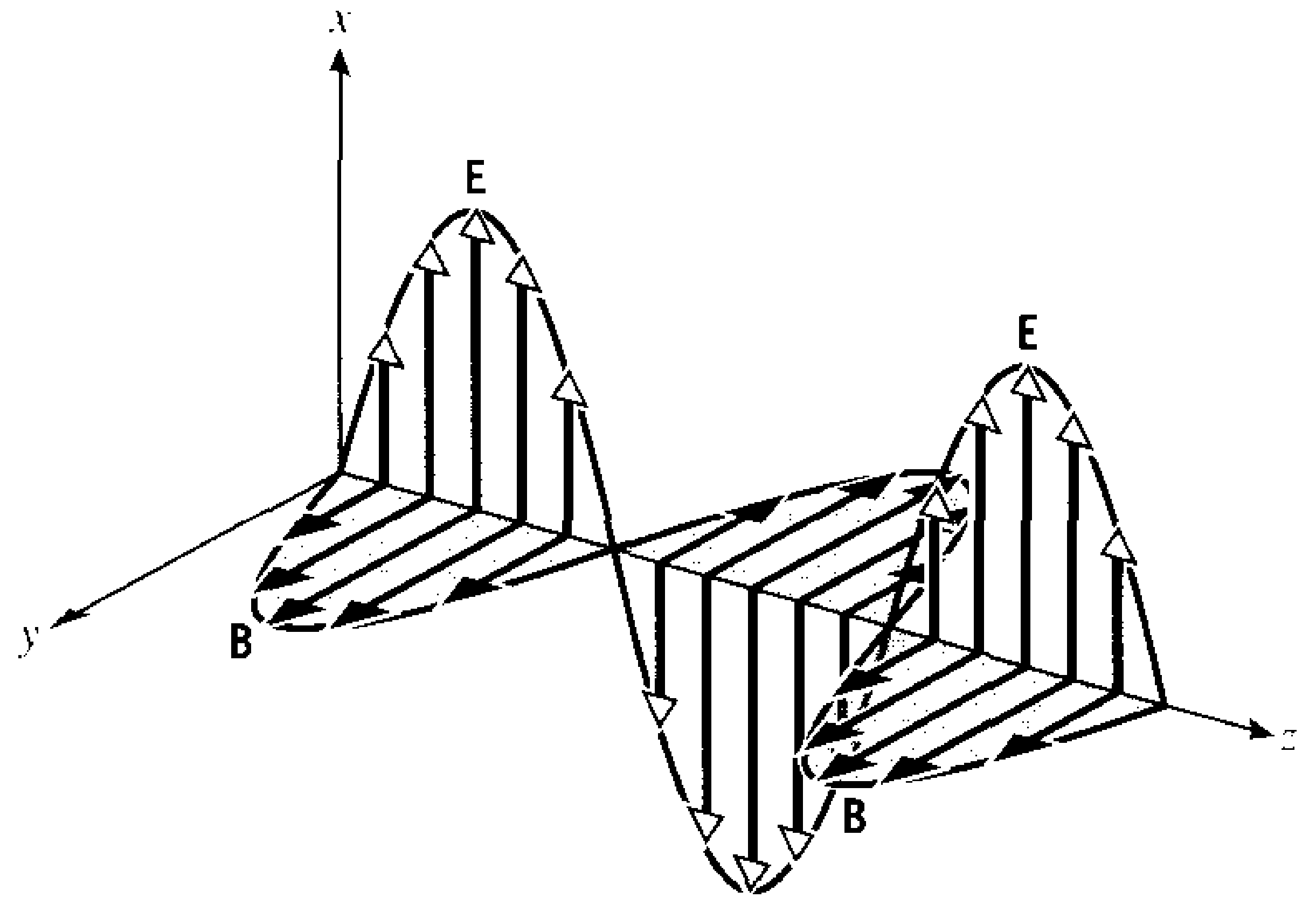


FIGURE 1-5 The electromagnetic wave.
The wave consists of oscillating electric and magnetic fields that are perpendicular to each other and to the direction of propagation (z direction).

Consider the case where the electric field has the simple form

$$\mathbf{E} = E_0 \cos(kz - \omega t)\mathbf{i}_x. \qquad (1.18)$$

A wave of this form is called a *plane wave* because the magnitudes of the electric and magnetic fields do not depend on the x or y coordinates. When we substitute this expression for the electric field into the wave equation, we get a relationship between the constants k, ω and the wave speed c:

$$c = \frac{\omega}{k}. \qquad (1.19)$$

The constant ω is called the *angular frequency* and the constant k is called the *wave number*. When we make the transformation,

$$z \to z + \frac{2\pi}{k}, \qquad (1.20)$$

the electric field does not change because

$$\cos(kz - \omega t) = \cos(kz - \omega t + 2\pi). \qquad (1.21)$$

The amplitude of the electric field is periodic in the coordinate z. The *wavelength* (λ) of the wave is defined to be

$$\lambda \equiv \frac{2\pi}{k}, \qquad (1.22)$$

so that the transformation

$$z \to z + \lambda, \qquad (1.23)$$

leaves the electric field unchanged. Similarly if we make the transformation

$$t \to t + \frac{2\pi}{\omega}, \qquad (1.24)$$

the electric field is also unchanged. The *period* (T) of the wave is defined to be

$$T \equiv \frac{2\pi}{\omega}, \qquad (1.25)$$

and the inverse of the period is defined to be the *frequency* (f)

$$f \equiv \frac{1}{T} = \frac{\omega}{2\pi}. \qquad (1.26)$$

From the relationship (1.19) between k and ω, we have

$$c = \lambda f. \qquad (1.27)$$

The speed of the wave propagation is equal to the wavelength multiplied by the frequency. Electromagnetic waves may have a wavelength of any size. The electromagnetic spectrum is summarized in Table 1-2.

The speed of a plane wave is equal to the wavelength times the frequency,

$$c = \lambda f.$$

Using the superposition principle, an arbitrary wave may always be constructed from plane waves of different amplitudes (A_n) and wave numbers (k_n):

$$F(z - ct) = \sum_{n=0}^{\infty} A_n \cos(k_n z - ck_n t). \qquad (1.28)$$

This expression is also a solution of the wave equation.

1-3 LOOKING INSIDE THE ATOM: ELECTRONS AND A NUCLEUS

X Rays, Alpha and Beta Particles

At the end of 1895, Wilhelm Röntgen discovered *x rays*, a mysterious radiation that penetrated matter. The discovery of Röntgen started a revolution in physics. (Röntgen's discovery will be taken up in more detail in Chapter 3.) In 1896, Antoine Henri Becquerel set out to investigate if the phenomena of fluorescence and phosphorescence (light

TABLE 1-2
THE ELECTROMAGNETIC SPECTRUM.

There is no convention for the exact wavelengths of the boundaries.

Wavelength	**Name**
less than 10^{12} m	γ ray
10^{12} to 10^{9} m	x ray
10^{9} to 4×10^{7} m	ultraviolet
4×10^{7} m to 7×10^{7} m	light
7×10^{7} m to 10^{3} m	infrared
10^{3} m to 0.1 m	microwave
0.1 m to 10^{3} m	radio
greater than 10^{3} m	ultra low frequency

emission by certain substances when exposed to radiation) produced x rays. In this investigation Becquerel discovered a new type of radiation. Marie and Pierre Curie investigated the properties of the Becquerel radiation and called the phenomenon *radioactivity*. Natural radioactivity is the spontaneous emission of radiation from certain heavy elements such as uranium. (Radioactivity is discussed in detail in Chapter 11.) This natural radiation was discovered to be *quantized* in the form of particles. Ernest Rutherford observed that the particles emitted from atoms were of two types: *alpha* particles (α), which did not penetrate matter, and *beta* particles (β), which easily penetrated matter (see Figure 1-6). The α and β particles were destined to play a crucial role in the understanding of the atom. These particles were used to probe the structure of the atom. Furthermore, the quest for the understanding of their existence would lead to the discovery of two new forces!

Discovery of the Electron

A type of radiation, called *cathode rays*, was observed to be emitted from metallic surfaces when voltage was applied. At the end of the nineteenth century there was much theoretical speculation about the fundamental properties of the cathode rays. One school of thought held the belief that cathode rays were particles. The main evidence for the particle hypothesis was the observation that cathode rays were deflected by magnetic fields. The main obstacle to this interpretation was the lack of observation of the deflection of cathode rays by electric fields. The other school of thought held the belief that cathode rays were a wave phenomenon dependent on the medium of space (*ether* or *aether*). The wave interpretation was supported by the observation that cathode rays could pass through metal foils without deflection. It is interesting to note that physicists were troubled by the apparent contradiction of both a particle and a wave interpretation. This important issue would arise again in 30 years.

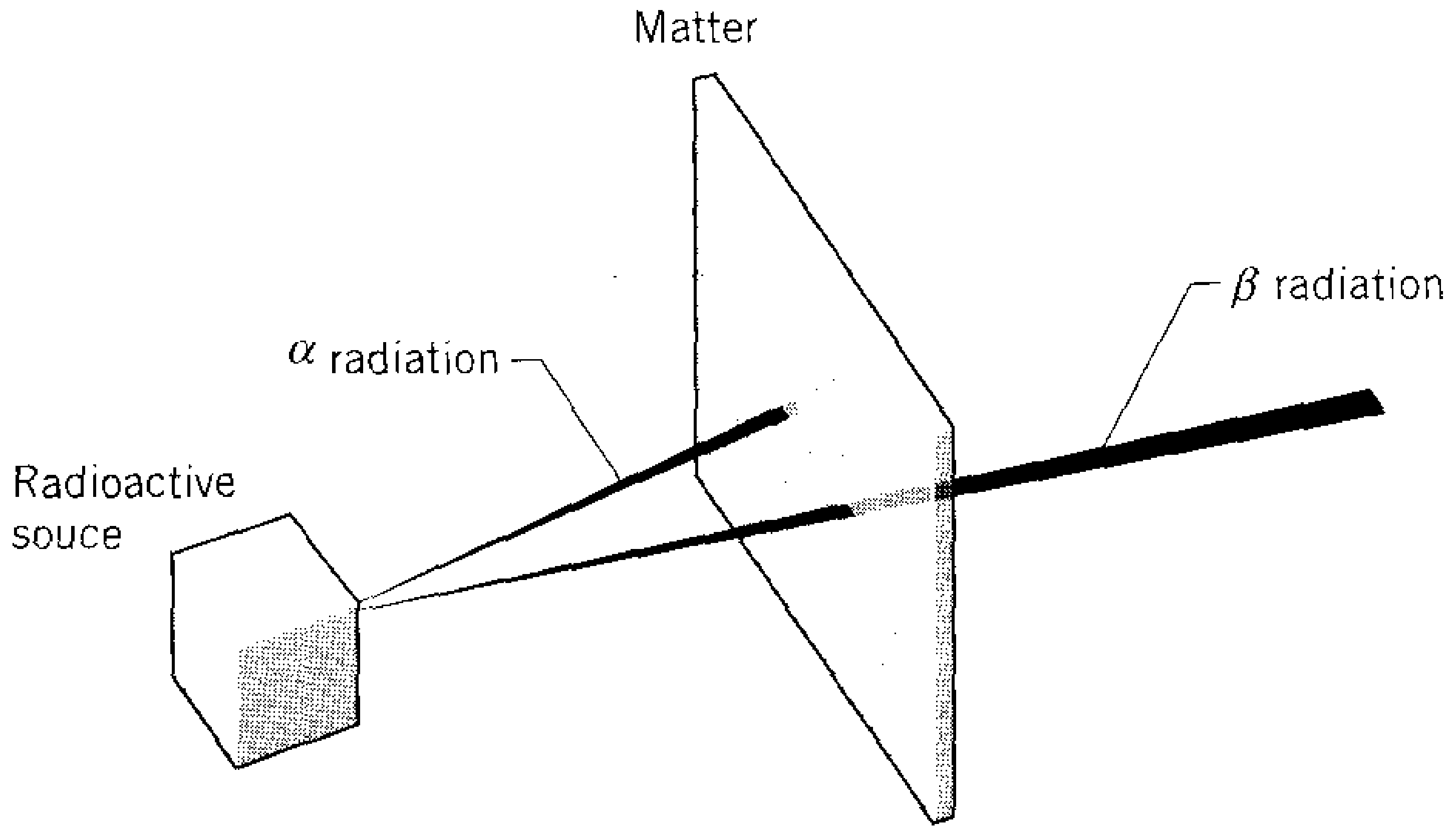


FIGURE 1-6 Rutherford's classification of the radiation discovered by Becquerel.
The α particle does not penetrate matter, whereas the β particle readily penetrates matter.

In 1897, Joseph John Thomson performed a definitive set of experiments that proved that cathode rays had a particle behavior. The situation was summed up by Thomson in the introduction of the paper reporting his results:

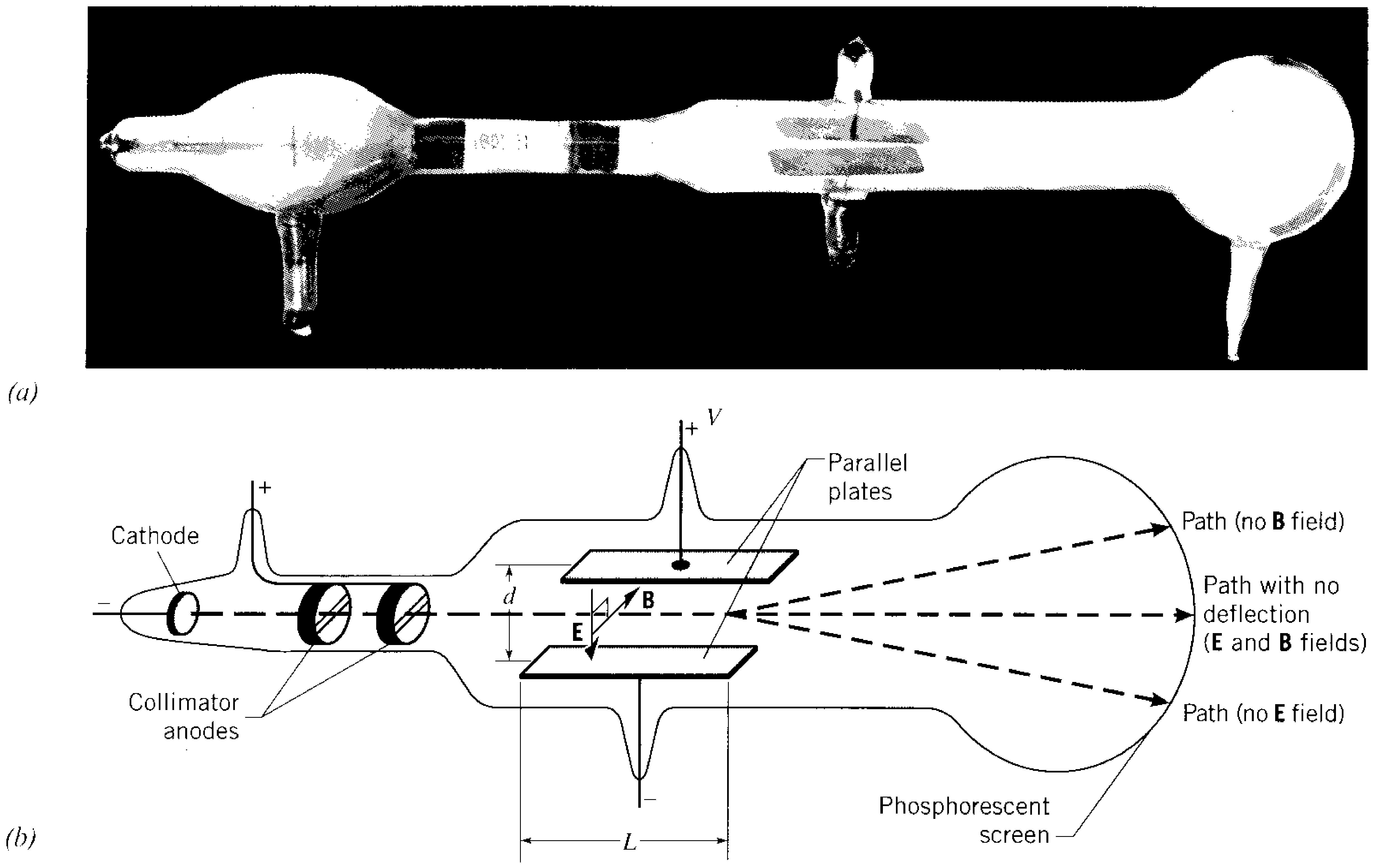
> *The experiments discussed in this paper were undertaken in the hope of gaining some information as to the nature of Cathode Rays. The most diverse opinions are held as to these rays; according to the almost unanimous opinion of German physicists they are due to some process in the aether to which—inasmuch as in a uniform magnetic field their course is circular and not rectilinear—no phenomenon hitherto observed is analogous; another view of these rays is that, so far from being wholly aetherial, they are in fact wholly material, with negative electricity. It would seem at first sight that it ought not to be difficult to discriminate between views so different, yet experience shows that this is not the case, as amongst the physicists who have deeply studied the subject can be found supporters of either theory.*

The Experiment of J. J. Thomson

The key to the success of the Thomson experiment was the development of the technique necessary to observe the deflection of cathode rays in an electric field. This led to the interpretation of cathode rays as charged particles, commonly known as *electrons*. In his experiment, Thomson accelerated electrons in an electric field and measured the curvature of their trajectories in a magnetic field, clearly demonstrating that they were particles with a negative electric charge. The apparatus developed by Thomson is called a *mass spectrometer*. Thomson used his spectrometer to measure the charge-to-mass ratio of the electron.

The Thomson spectrometer is illustrated in Figure 1-7. A stream of electrons emitted from a cathode passed through collimators into a region of two parallel plates of length L, separated by a distance d. A voltage (V) was applied to the plates, creating an electric field (E):

$$E = \frac{V}{d}. \tag{1.29}$$



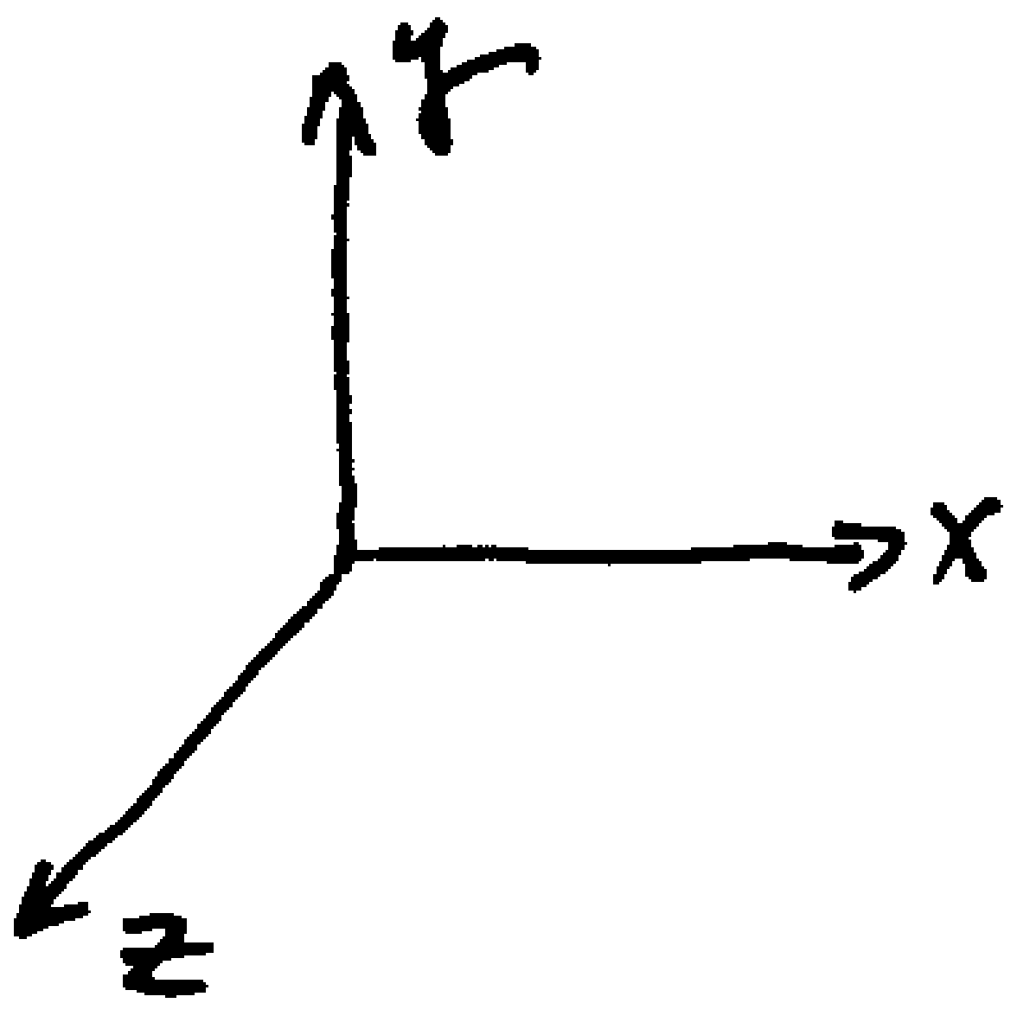


FIGURE 1-7 Spectrometer used by J. J. Thomson to measure the charge-to-mass ratio of the electron. (*a*) Photograph courtesy of Science Museum Library, London. (*b*) Schematic fashioned after J. J. Thomson, "Cathode Rays," *Phil. Mag.* **44,** 293 (1897).

The apparatus was enclosed in an evacuated glass tube to reduce collisions of the electrons with molecules in the air. The vacuum was important because collisions of the electrons with molecules produced ions (charged particles). The ions would collect on the plates and cancel the effect of the applied electric field! This is why the deflection of cathode rays by electric fields had escaped detection.

The acceleration (a) of the electron is in the y direction with a magnitude equal to the electric force divided by the electron mass (m). Taking the electron charge to be q, we have

$$a = \frac{F}{m} = \frac{qE}{m} = \frac{qV}{md}. \tag{1.30}$$

The time (t_p) that the electron spends between the plates is inversely proportional to the x component of the electron velocity (v_x):

$$t_p = \frac{L}{v_x}. \tag{1.31}$$

The y component of velocity (v_y) is the product of the acceleration (1.30) and the time (1.31),

$$v_y = at_p = \frac{qVL}{mdv_x}. \tag{1.32}$$

This gives the following expression for the charge-to-mass ratio (q/m) of the electron:

$$\frac{q}{m} = \frac{dv_y v_x}{VL}. \tag{1.33}$$

Since d, L and V, were known quantities, measurement of v_x and v_y would determine q/m. The position of the stream of electrons was measured on a phosphorescent screen. The ratio of v_x and v_y is a relatively easy quantity to measure because it is the tangent of the deflection angle (θ):

$$\frac{v_y}{v_x} = \tan\theta. \tag{1.34}$$

The expression for q/m, however, involves the product $v_y v_x$, and the electron velocity is not an easy quantity to measure directly. Thomson thought of a clever method to determine the electron velocity. Thomson put a magnetic field perpendicular to the electric field and adjusted it such that there was no deflection. In the case for no deflection, the electric and magnetic forces are balanced:

$$qE = qv_x B. \tag{1.35}$$

$$-e(\vec{v} \times \vec{B})$$

Thus, v_x is determined by measurement of E and B:

$$v_x = \frac{E}{B} = \frac{V}{dB}. \tag{1.36}$$

The expression for q/m (1.33) may be written as

$$\frac{q}{m} = \frac{d\left(\frac{v_y}{v_x}\right)v_x^2}{VL} = \frac{V \tan\theta}{dLB^2}, \tag{1.37}$$

where θ is the deflection angle with no magnetic field, and B is the magnetic field which produces no deflection. Thomson measured the quantities on the right-hand side of the expression for q/m (1.37). Thomson's data are plotted in Figure 1-8 in the form of $\tan\theta$ versus B^2L/E. Thomson's early result for the electron charge-to-mass ratio was about 10^{11} C/kg. This ratio was observed to be constant, that is, all cathode rays gave the same value. The value of q/m for the electron determined by Thomson was substantially smaller (more than three orders of magnitude) than the values of q/m determined by electrolysis, that is, q/m for the electron is much smaller than for ionized atoms. There were two extreme possibilities: (1) The electron charge is much smaller than the charge of an ionized atom, or (2) the electron mass is much smaller than the mass of an ionized atom (or both!). The ability of the electron to penetrate matter led Thomson to believe that the mass of the electron was much smaller than the mass of an atom.

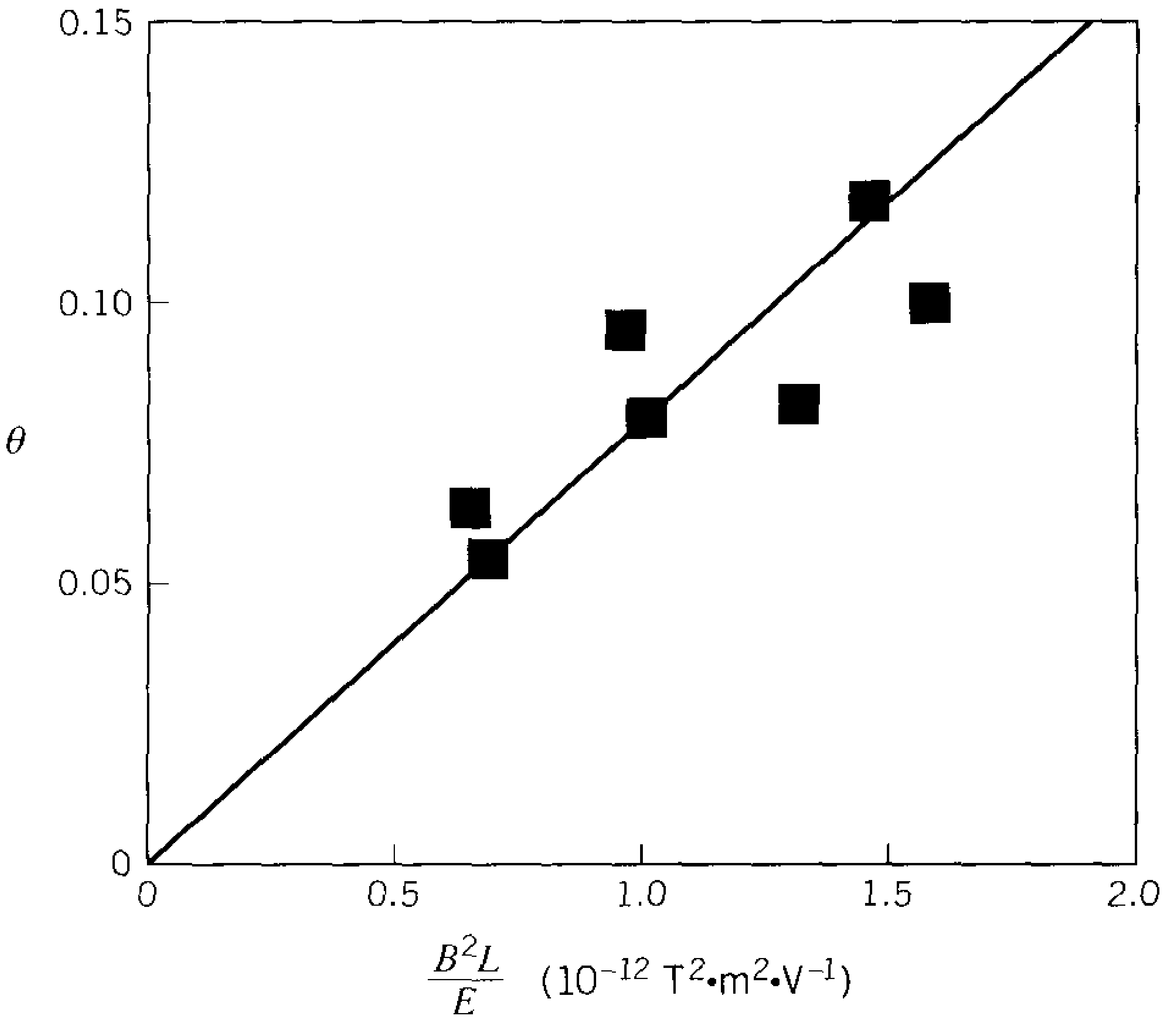


FIGURE 1-8 Thomson's data on the charge-to-mass ratio of the electron.
The data are expected to fall on a straight line passing through the origin. The slope gives q/m. Data are from J. J. Thomson, "Cathode Rays," *Phil. Mag.* **44**, 293 (1897).

Thomson's pioneering result was systematically low by about a factor of two due to the neglect of magnetic fields outside the deflecting plates in his spectrometer in measuring the deflection angle. Thomson knew of this effect and addressed it in his paper. An accurate measurement of the charge-to-mass ratio of the electron with the Thomson technique gives

$$\frac{q}{m} = 1.76 \times 10^{11} \text{ C/kg}. \tag{1.38}$$

EXAMPLE 1-4

The electric field in the Thomson spectrometer is set at 10^4 V/m and the deflection angle is observed to be 0.10 radians after passing through a distance of L = 0.050 m when there is no magnetic field. Calculate the speed of the electron.

SOLUTION:

First we calculate the magnetic field strength needed to produce no deflection,

$$B = \sqrt{\frac{E\tan\theta}{L\left(\frac{q}{m}\right)}} = \sqrt{\frac{(10^4 \text{ V/m})(0.1)}{(0.05\text{m})(1.76\times10^{11} \text{ C/kg})}} = 3.4\times10^{-4} \text{ T}.$$

The electron speed is the electric field divided by the magnetic field,

$$v = \frac{E}{B} = \frac{10^4 \text{ V/m}}{3.4\times10^{-4} \text{ T}} \approx 2.9\times10^7 \text{ m/s}. \quad \blacksquare$$

The Millikan Oil-Droplet Experiment

In 1909, Robert Millikan made the first accurate measurement of the electron charge. The experiment of Millikan is sketched in Figure 1-9. Tiny droplets of oil are sprayed between two conducting plates and viewed under a microscope. The oil droplets fall toward the earth due to gravity, and they experience a frictional *drag* force that is proportional to the speed of the droplet. The drag force is due to the collisions of the oil droplet with the air molecules. The droplet quickly reaches the equilibrium condition where the net force on the droplet is zero. The acceleration of the droplet is then equal to zero and the droplet falls with a constant speed called the *terminal* speed (v_T). The gravitational force (mg) is balanced by the drag force (bv_T)

$$mg = bv_T. \tag{1.39}$$

The coefficient b in the drag force term is directly proportional to the radius of the droplet (R). The coefficient b also depends on how "sticky" or *viscous* the air is. We write the drag force constant as

$$b = 6\pi\eta R, \tag{1.40}$$

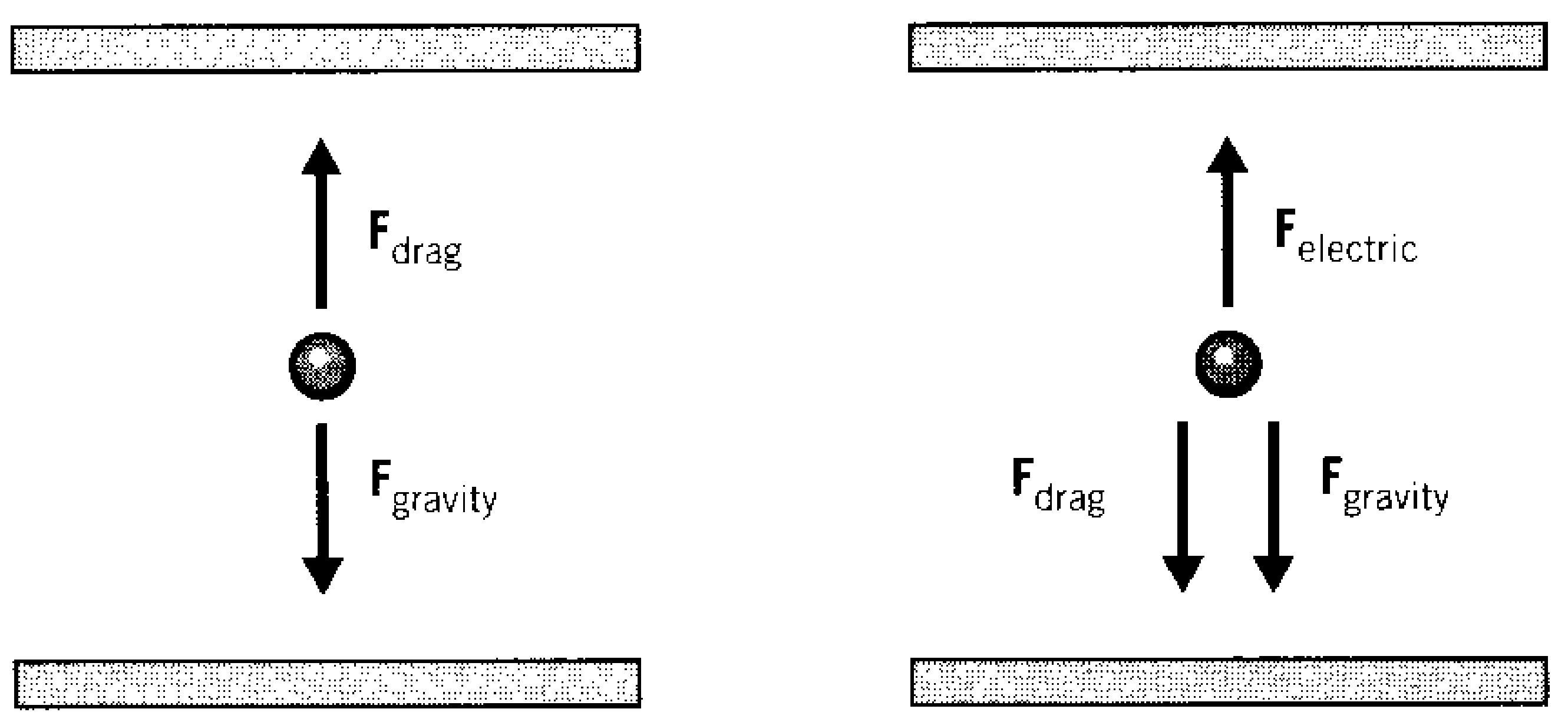
where η is the coefficient of viscosity of air. This result is known as Stokes's law. The mass of the droplet (m) and density of the oil (ρ) are related by

$$\rho = \frac{3m}{4\pi R^3}. \tag{1.41}$$

Eliminating b and m in the force equation (1.39), we may solve for the radius of the droplet:

$$R = \sqrt{\frac{9 v_T \eta}{2 g\rho}}. \tag{1.42}$$

Measurement of the terminal speed of the droplet together with knowledge of the density of the oil and coefficient of viscosity of air gives the size of the droplet. The typical size of an oil droplet in the Millikan experiment was 3×10^{-6} m (3 μm).



(*a*) No field (*b*) Electric field on

FIGURE 1-9 The Millikan oil-droplet experiment. (*a*) An oil droplet in free fall reaches a terminal speed, with the frictional drag force opposing the force of gravity. (*b*) An electric field is present, giving an upward force on a charged oil droplet that overcomes the force of gravity. The oil droplet reaches a terminal speed with the electric force opposing both the drag force and gravity.

In the process of spraying the oil, individual droplets may acquire one or more excess electrons. When an electric field is switched on, there is an electrical force on a charged droplet. If the electric field is directed downward, then the electrical force on the droplet is upward, against the force of gravity. If the electric force is stronger than the gravitational force, then the droplet moves upward and the drag force is now directed downward. The droplet reaches a terminal velocity (v_E) given by

$$qE = mg + bv_E. \tag{1.43}$$

Eliminating b from the force equations (1.39) and (1.43), we have

$$q = \frac{mg\left(1 + \frac{v_E}{v_T}\right)}{E}. \tag{1.44}$$

Since the size of the droplet (1.42) can be calculated and the density of the oil is readily measured, the mass of the droplet is also known. Thus, measurement of the two terminal speeds, v_T with no field and v_E with an electric field E applied, gives the electric charge on the droplet.

Millikan measured the trajectory of a *single* droplet over a long period (several minutes), turning on the electric field when the droplet was near the bottom plate and turning off the electric field when the droplet was near the top plate. By repeated measurements of the droplet speeds v_E and v_T, Millikan made a determination of the charge (1.44) on the oil droplet. The result of numerous measurements by Millikan was that the charge on the droplets was always an integer multiple of 1.6×10^{-19} C. Furthermore, Millikan observed that occasionally a droplet would gain or lose a charge of an integer multiple of 1.6×10^{-19} C as it drifted through the air. Millikan deduced that the change in charge (Δq) on the droplet was caused by collisions of the droplet with air molecules, resulting in the gain or loss of one or more electrons. Thus, Millikan determined that the magnitude of the charge of the electron was 1.6×10^{19} C. The data of Millikan from a single droplet are shown in Figure 1-10, where we plot the fractional deviation of Δq from the nearest integer multiple of the electron charge (e).

On the numerical value of the electron charge, Millikan wrote:

> *Perhaps these numbers have little significance to the general reader who is familiar with no electrical units save those in which his monthly light bills are rendered. If these latter seem excessive, it may be cheering to reflect that the number of electrons contained in the quantity of electricity which courses every second through a common sixteen-candle-power electric-lamp filament, and for which we pay 1/100,000 of 1 cent, is so large that if all the two and one-half million inhabitants of Chicago were to begin to count out these electrons and were to keep on counting them out each at the rate of two a second, and if no one of them were ever to stop to eat, sleep, or die, it would take them just twenty thousand years to finish the task.*

The result of Millikan marked the discovery of *charge quantization*. Charge is an intrinsic property of the electron. It is not possible to remove the charge from an electron; there is no such thing as an electron without its charge. The fundamental unit of charge in modern physics is called *e*. The electron is defined to have an electric charge of *minus e*:

$$\text{electron charge} = -e\,. \tag{1.45}$$

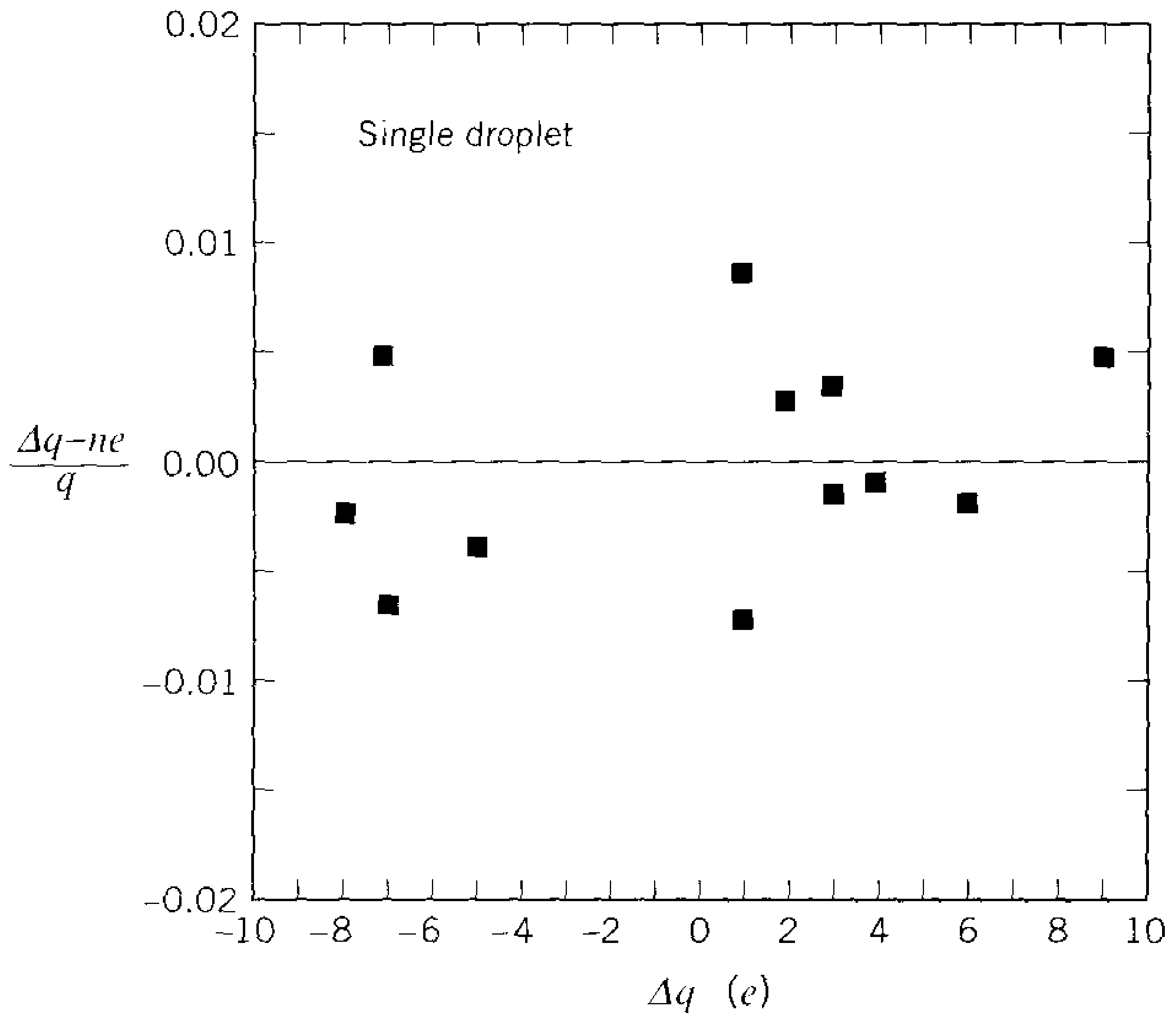


FIGURE 1-10 Data of Millikan.
The terminal speeds (v_T and v_E) of a single oil droplet were measured over a time interval of 45 minutes and the electric charge was computed. Occasionally the charge of the droplet changed by collision with air molecules as evidenced by a change in v_E. The data show the change in charge of the droplet (Δq) is equal to an integer (n) times a fundamental charge (e). Data from R.A. Millikan, *Electrons (+ and –), Protons, Photons, Mesotrons, and Cosmic Rays*, University of Chicago Press (1947).

Millikan's measurement of *e* was systematically low by about 0.4% due to inaccurate knowledge of the coefficient of viscosity. Accurate measurement of the value *e* gives

$$e = 1.602 \times 10^{-19}\ \text{C}. \tag{1.46}$$

Particles that are *electrically* attracted to an electron are assigned a positive charge, and particles that are electrically repelled by an electron are assigned a negative charge. Particles that are neither attracted nor repelled electrically are assigned a charge of zero. All free particles are observed to have values of electric charge (q) equal to an integer times the fundamental charge e:

$$q = ne\,, \tag{1.47}$$

where $n = \ldots -3, -2, -1, 0, 1, 2, 3 \ldots$ The integer n is called the electric charge *quantum number*. A free particle has never been observed with a charge unequal to an integer times the electron charge.

The experimental results of Thomson and Millikan may be combined to yield the mass of the electron:

$$m = q\left(\frac{m}{q}\right) = 9.11 \times 10^{-31}\ \text{kg}\,. \tag{1.48}$$

The electron mass is much smaller than the mass of the hydrogen atom (see Example 1-1).

The Wilson Cloud Chamber

In 1906, Charles T. R. Wilson, a student of Thomson's, made a brilliant discovery on the detection of charged particles. When a charged particle such as an electron passes through any material, there is an electromagnetic interaction between the charged particle and the electrons in the atoms of the material. This interaction is strong enough to remove electrons or *ionize* atoms in the material. The ionization occurs along the trajectory of the moving charged particle.

Imagine that the material being ionized by an energetic incoming electron is damp air. If the air is suddenly expanded, droplets of condensation form. Wilson's discovery was this: The droplets of condensation form around the ions! The droplets may be photographed, revealing a picture of the ionization trail created by the electron. This device is called a *cloud chamber*.

Sudden Cooling

The cloud chamber was an extremely important tool in discovering particles and measuring their properties. The amount of ionization observed in the cloud chamber depends on the speed of the charged particle. A particle with a lower speed spends more time in the vicinity of individual atoms and produces more ionization. Thus, the density of droplets in the cloud chamber gives information on the particle speed. If the cloud chamber is placed in a magnetic field, the curvature of the particle trajectory may be measured to determine the particle momentum. If the speed and momentum are known, the mass of the particle can be calculated.

The charge-to-mass ratio of the β particle discovered by Becquerel was determined to be the same as that of the electron. The β particle *is* the electron! Wilson's discovery allows us to see the trajectory of an electron. An early picture of an electron track in a cloud chamber made by C. T. R. Wilson is shown in Figure 1-11. The source of the electron is the radioactive decay of radium, the β particles discovered by Becquerel.

Discovery of the Nucleus

One early hint that atoms might have structure (i.e., contain other particles) was the fact that there are so many elements. If each atom was a fundamental particle, then there would be more than 100 types of fundamental particles. This could have been the case, but physicists were suspicious of having so many fundamental particles in nature. Furthermore, groups of atoms (see Figure 1-1) have similar properties. If atoms contain electrons and the net charge of atoms is zero, the atoms must also contain positive charge.

In 1912, Ernest Rutherford and his associates discovered that the positive charge of the atom is concentrated in a *nucleus*. (The Rutherford experiment is discussed in detail in Chapter 6.) Rutherford discovered the nucleus by experimenting with the α particles discovered by Becquerel. The charge of the α particle was determined to be $2e$ and the mass of the α particle was determined to be about four times the mass of the hydrogen atom.

Rutherford directed α particles through thin foils of material and observed that they are occasionally scattered at very large angles ($\theta > \pi/2$). Since the α particle has a large mass compared to the electron mass, collisions with electrons cannot produce large scattering angles. Rutherford ingeniously deduced that the large scattering angles are the result of a large electric force between the α particle and the nucleus and that the large force is

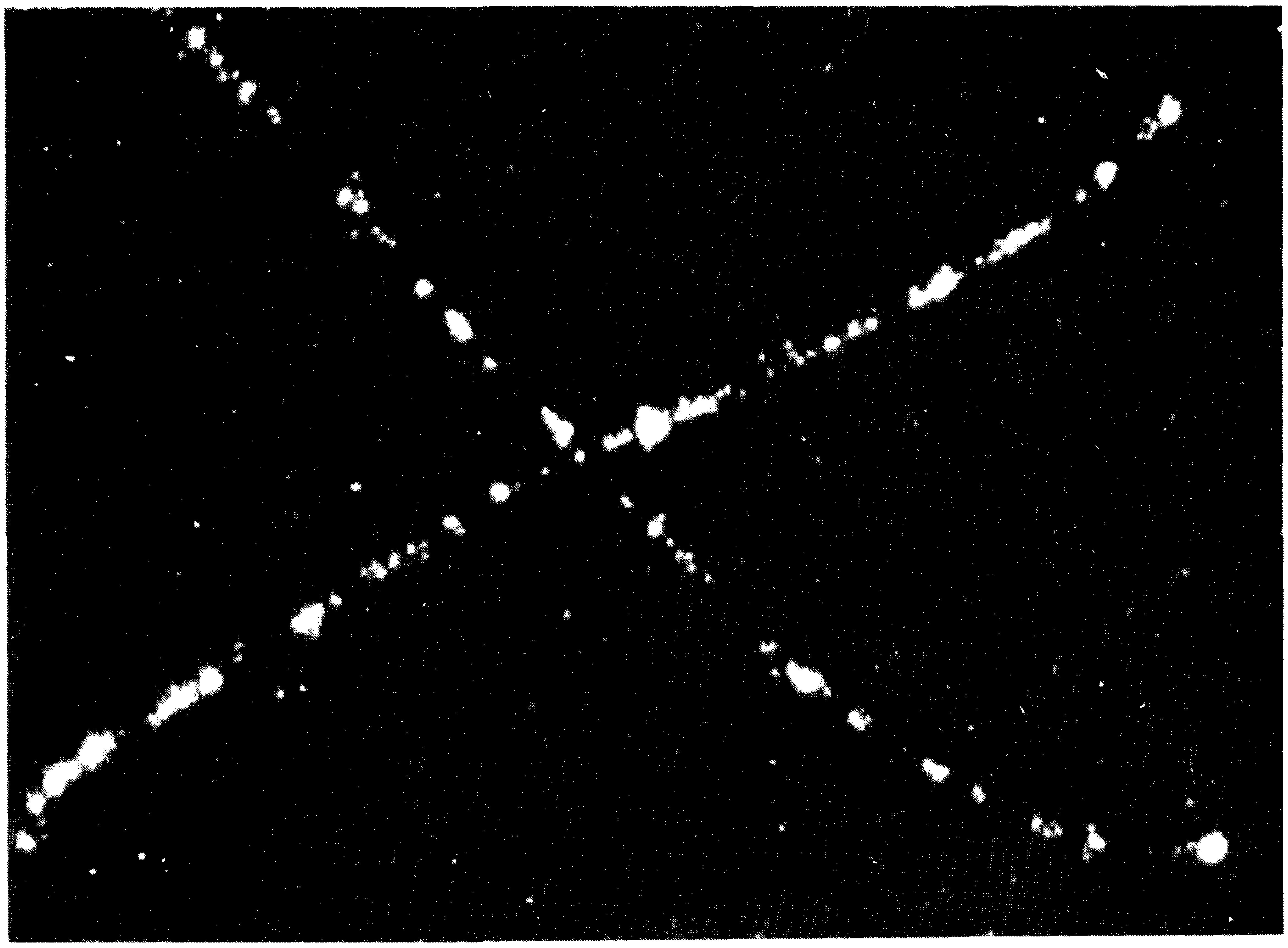


FIGURE 1-11 Electron tracks in a cloud chamber made by C. T. R. Wilson.
Courtesy Science Museum Library, London.

Sudden
Cooling

possible only at short distances, which in turn is possible only if the nucleus is concentrated at a "point."

The experiments of Rutherford also showed that the nucleus contains almost all the mass of the atom and that each element has a nucleus with a unique electric charge. The charge-to-mass ratio of the helium nucleus was measured to be the same as that of the α particle. The α particle *is* the helium nucleus.

The nucleus and the electrons are attracted to each other by the electromagnetic force. The strength of this force determines the size of atoms. The size of an atom is given by the typical distance between an outer electron and the nucleus. The stronger the force, the closer the electrons are to the nucleus, on the average. *What keeps electrons from being attracted all the way to the nucleus?* This question baffled physicists for a number of years. The answer will be explained in Chapter 5. The atom does not collapse because the electron behaves as a wave. In an atom, an electron cannot be localized to infinite precision. There are two competing processes in the electron–nucleus interaction: The electromagnetic force is pulling the electron to the nucleus and the electron is waving itself away.

Experiments like those of Thomson show us that electrons are *identical* particles in each atom. The measurement of the charge of the nucleus shows that there are different numbers of electrons in each type of atom. In discovering this important fact, we have made a great step in understanding the structure of matter, because the chemical properties of the elements may be explained by the *number* of electrons in each atom. The properties of carbon differ greatly from the properties of oxygen because the carbon atom has 6 electrons and oxygen has 8, but the electrons in the two atoms are identical particles. We have also raised a fundamental question because the nucleus is different for each element. *Why are there so many different nuclei?*

1-4 LOOKING INSIDE THE NUCLEUS: PROTONS AND NEUTRONS

One early hint that the nucleus is not a fundamental constituent of matter is that there are so many of them! The nucleus of the lightest atom (hydrogen) is called the *proton*. The charge of the proton is measured to be e. A new particle with zero electric charge was discovered by bombarding beryllium atoms with α particles. James Chadwick showed that the new particle, called the *neutron*, had mass nearly equal to that of the proton. (The discovery of the neutron and the measurement of its properties are described in Chapter 11.)

Through detailed study of many nuclear interactions, neutrons and protons were found to be components of every nucleus. The nucleus of each atom is made up of identical protons and identical neutrons. Nuclei of the elements differ only by their *numbers* of protons and neutrons. The elements are classified by the number of protons (Z) and the total number of protons plus neutrons (A). The mass of an atom is dominated by the neutrons and protons. This is why the atomic mass is very nearly proportional to an integer (A).

We should make a distinction between the atomic mass number (an integer) and the slightly different atomic mass for which we use the same symbol (A). The atomic mass number is defined to be the number of neutrons plus protons in the nucleus. The atomic mass is defined to be the number of grams that correspond to N_A atoms. The atomic mass is not exactly equal to the number of neutrons plus protons for several reasons: (1) The electrons have some mass, (2) the proton and neutron have a small mass difference, and (3) the neutrons and protons interact with each other. These effects have a relatively small influence on the atomic mass. For example, the atomic mass number of the isotope of oxygen that makes up 99.8% of its natural abundance is equal to 16 (eight neutrons and eight protons). The atomic mass of this atom is equal to 15.994915, very nearly equal to 16.

It is possible for the nuclei of two atoms to have the same number of protons and different numbers of neutrons. Two atoms with the same atomic number (Z) but different atomic mass number (A) are called isotopes. Obviously, the atomic masses of isotopes differ. The value of A listed in a periodic table is usually the weighted average of the isotopes as they occur in nature. For example, carbon may be seen listed as $A = 12.01$, reflecting the fact that the most common isotope of carbon is $A = 12$, but there is a tiny percentage of the isotopes with $A = 13$ and $A = 14$.

The composition of atoms is explained by the electrical attraction of a positively charged nucleus and negatively charged electrons. A force other than the electric force is needed to explain the attraction of the protons and neutrons to one another in the formation of nuclei. This force is called the *strong force*. The strong force between any combination of two neutrons or protons is attractive and of equal strength. The strong force between an electron and a proton or neutron is zero. In the same manner that the strength of the electromagnetic force determines the size of atoms, the strength of the strong force determines the size of the nucleus. All nuclei are the same order of magnitude in size. The approximate diameter of a proton (d_{proton}) is 2 femtometer (fm):

$$d_{\text{proton}} \approx 2\times 10^{-15}\ \text{m} = 2\ \ \text{fm}. \tag{1.49}$$

1-5 MASS AND BINDING ENERGY

Energy Units

The energy units of modern physics are derived from the unit of potential difference in electricity, the volt (V). The definition of the volt is

$$1\ \text{V} \equiv 1\ \text{J/C}. \tag{1.50}$$

The unit of energy is called the electronvolt (eV). One electronvolt is defined to be the amount of kinetic energy that an electron acquires when it is accelerated through a potential difference of 1 volt. Since we have defined the magnitude of the electron charge to be e, the joule and the electronvolt are related by

$$\begin{aligned} 1\ \text{J} &= (1\ \text{V})(1\ \text{C})\left(\frac{e}{1.602\times 10^{-19}\ \text{C}}\right) \\ &= \frac{e(1\ \text{V})}{1.602\times 10^{-19}} = \frac{1\ \text{eV}}{1.602\times 10^{-19}}, \end{aligned} \tag{1.51}$$

or

$$1\text{eV} = 1.602\times 10^{-19}\ \text{J}. \tag{1.52}$$

The electric force on a particle depends on the charge of the particle but not on its mass. Thus, a proton accelerated through 1 V also acquires a kinetic energy of 1 eV.

By definition of the electron charge (1.46) and the electronvolt (1.52),

$$\frac{1\ e}{1\ \text{C}} = \frac{1\ \text{eV}}{1\ \text{J}}. \tag{1.53}$$

EXAMPLE 1-5

The electric force constant k in Coulomb's law is measured to be 8.99×10^9 J·m/C^2. Calculate the quantity ke^2 in units of J·m and eV·nm.

SOLUTION:

We have

$$\begin{aligned} ke^2 &= \left(8.99\times 10^{9}\ \text{J}\cdot\text{m/C}^2\right)\left(1.60\times 10^{-19}\ \text{C}\right)^2 \\ &= 2.30\times 10^{-28}\ \text{J}\cdot\text{m}, \end{aligned}$$

and

$$ke^2 = \left(\frac{2.30\times 10^{-28}\ \text{J}\cdot\text{m}}{1.60\times 10^{-19}\ \text{J/eV}}\right)\left(\frac{10^{9}\ \text{nm}}{\text{m}}\right) = 1.44\ \text{eV}\cdot\text{nm}.\ ■$$

EXAMPLE 1-6

Calculate the strength of the electric field at a distance of 0.1 nm from a proton.

SOLUTION:

The electric field is

$$\begin{aligned} E &= \frac{ke}{r^2} = \frac{ke^2}{er^2} = \frac{1.44\ \text{eV}\cdot\text{nm}}{e(0.1\ \text{nm})^2} \\ &= 144\ \text{V/nm} = 1.44\times 10^{11}\ \text{V/m}. \end{aligned}$$

The electric fields are gigantic on atomic scales compared to fields accessible in the laboratory! ■

EXAMPLE 1-7

The magnetic field at the center of a current-loop is $(2\pi kI)/(c^2R)$, where I is the current in the loop and R is the radius of the loop. If the hydrogen atom is modeled as an electron moving in a circular orbit with $R = 0.1$ nm at a speed of 3×10^6 m/s, calculate the strength of the magnetic field inside the atom.

SOLUTION:

A moving charge constitutes a current ($I = dq/dt$). A charge moving in a circle of radius R with speed v makes one revolution in the time

$$T = \frac{2\pi R}{v}.$$

The corresponding current is

$$I = \frac{e}{T} = \frac{ve}{2\pi R}.$$

The magnetic field is

$$\begin{aligned} B &= \frac{2\pi kI}{c^2 R} = \frac{ke^2 v}{ec^2 R^2} \\ &= \frac{\left(1.44\times 10^{-9}\ \text{eV}\cdot\text{m}\right)\left(3\times 10^{6}\ \text{m/s}\right)}{(e)\left(3\times 10^{8}\ \text{m/s}\right)^2\left(10^{-10}\ \text{m}\right)^2} \approx 5\ \ \text{T}. \end{aligned}$$

This is a large magnetic field by laboratory standards.

■

The electromagnetic force constant (see Example 1-5),

$$ke^2 = 1.44 \text{ eV} \cdot \text{nm}, \qquad (1.54)$$

is worth remembering because it specifies the strength of the electric force and is often used in calculations. The strength of the electric force on the atomic scale is given directly by the numerical value of ke^2. An electron and proton separated by a fraction of a nanometer will require a few electronvolts of energy in order to separate them. The electronvolt is a convenient unit because it is the characteristic energy scale of electrons in atoms (see Table 1-3).

> The strength of the electric force (F) between an electron and a proton that are separated by a distance r is
>
> $$F = \frac{ke^2}{r^2} = \frac{1.44 \text{ eV} \cdot \text{nm}}{r^2}.$$

Mass Energy

Consider the β particles (electrons) from the spontaneous decay of a heavy nuclei first observed by Becquerel. These electrons did not exist before the nuclei decayed! The electrons are *created* in the decay process! The electrons are produced with a typical kinetic energy of an MeV. *Where does this energy come from?*

TABLE 1-3
ENERGY UNITS IN MODERN PHYSICS.

Energy	Physical Interpretation
eV	Energy scale of the outer electrons in atoms
keV ≡ 10^3 eV	Energy scale of the inner electrons in heavy atoms
MeV ≡ 10^6 eV	Energy scale of neutrons and protons inside nuclei
GeV ≡ 10^9 eV	Energy scale of quarks inside protons
TeV ≡ 10^{12} eV	Energy scale to be studied by the next generation of particle physics experiments

The energy that the electron acquires was stored in the form of *mass energy* of the decaying nucleus. The *total energy* (E) of any particle is defined to be the sum of two parts: energy due to motion, called the *kinetic energy* (E_k), and energy stored as mass, called the *mass energy* (E_0). Energy is defined in this manner,

$$E \equiv E_k + E_0, \qquad (1.55)$$

because this quantity is observed to be conserved in all particle interactions. Einstein was the first to deduce that the mass energy of a particle is equal to the mass (m) times the speed of light squared,

$$E_0 = mc^2. \qquad (1.56)$$

The cornerstone of Einstein's theory is that the speed of light is an absolute constant that does not depend on the motion of the source. This is verified by experiment. (The theory of special relativity is the subject of Chapter 4.) The speed of light (c) is measured to be

$$c \approx 3.00 \times 10^8 \text{ m/s}. \qquad (1.57)$$

The electron mass energy is

$$\begin{aligned} E_0 &= mc^2 \\ &\approx \left(9.11\times10^{-31} \text{ kg}\right)\left(3.00\times10^8 \text{ m/s}\right)^2 \\ &\quad \times\left(\frac{1 \text{eV}}{1.60\times10^{-19} \text{ J}}\right) \\ \underline{m_e c^2} &\underline{\approx 5.11\times10^5 \text{ eV} = 0.511 \text{ MeV}.} \end{aligned} \qquad (1.58)$$

Similarly, the proton mass energy is

$$m_p c^2 : E_0 \approx 938 \text{ MeV}. \qquad (1.59)$$

A scale of energies found in nature together with their mass equivalents is shown in Figure 1-12.

> The mass energy of a particle (E_0) is the mass times the speed of light squared,
>
> $$E_0 = mc^2.$$

EXAMPLE 1-8

Calculate the amount of energy stored in 1 kg of matter.

SOLUTION:

The amount of energy is

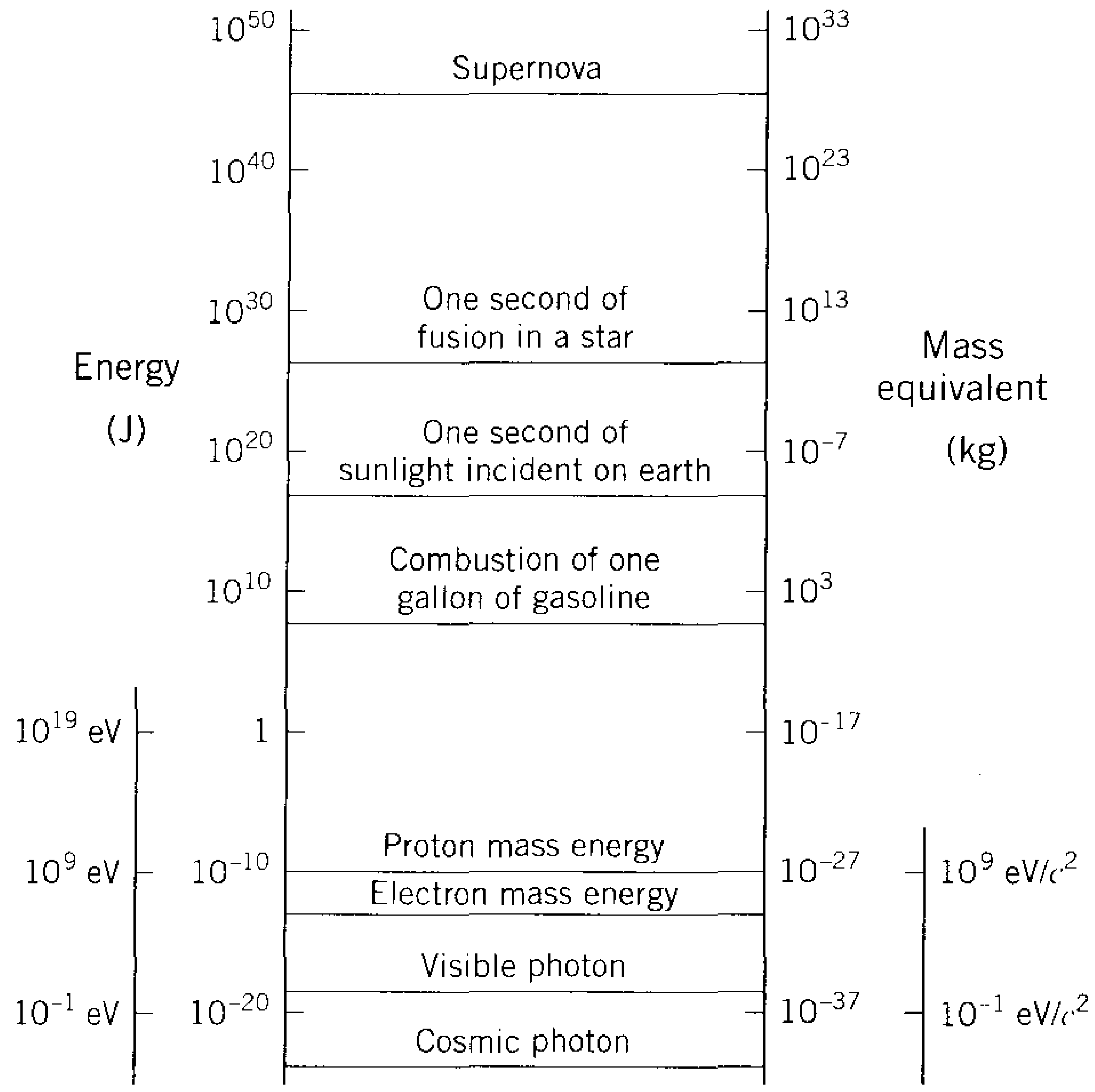


FIGURE 1-12 Energies in our universe and their equivalent masses.

$$E_0 = mc^2 = (1\text{ kg})(3.00\times 10^8\text{ m/s})^2 = 9\times 10^{16}\text{ J}.$$

This is an enormous amount of energy! If this energy could be released and harnessed, it could provide a kilowatt of power for about 3 million years! *Just how much energy is this?* At a cost of 10 cents per kilowatt-hour, the energy stored in one kilogram of dirt is worth about 2.5 billion dollars! ■

Most of the energy in the universe is stored in the form of mass energy. This was not always the case. About 10 billion years ago, most of the energy of the universe was in the form of kinetic energy. The physics of the early universe is discussed in Chapter 19.

The fundamental relationship between mass energy and mass of a particle (1.56) invites the use of a particularly useful mass unit, the MeV/c^2. By defining this natural mass unit, it saves us the trouble of dividing by the speed of light squared. It can be a tremendous convenience not to have to do the division. For more massive particles, we define the analogous mass unit, the GeV/c^2.

EXAMPLE 1-9

Calculate the electron mass in units of MeV/c^2.

SOLUTION:

The mass energy of the electron is

$$E_0 = 0.511\text{ MeV}.$$

Using the relationship between mass and mass energy, we have

$$m = \frac{E_0}{c^2} = 0.511\text{ MeV}/c^2.$$

That was easy! ■

EXAMPLE 1-10

Show explicitly that MeV/c^2 has units of mass by expressing it in kilograms.

SOLUTION:

We have

$$\begin{aligned} 1\text{ MeV} &= (10^6\text{ eV})(1.60\times 10^{-19}\text{ J/eV}) \\ &= 1.60\times 10^{-13}\text{ kg}\cdot\text{m}^2/\text{s}^2. \end{aligned}$$

Dividing by c^2, we get

$$\begin{aligned} 1\text{ MeV}/c^2 &= \frac{1.60\times 10^{-13}\text{ kg}\cdot\text{m}^2/\text{s}^2}{(3.00\times 10^8\text{ m/s})^2} \\ &= 1.78\times 10^{-30}\text{ kg}. \end{aligned}$$

■

EXAMPLE 1-11

The atomic number of hydrogen is $A = 1.0078$. Calculate the mass energy of the carbon atom ($A = 12$) in electronvolts.

SOLUTION:

The mass energy of the hydrogen atom is the sum of the masses of the electron and proton (neglecting the atomic binding energy):

$$m_{\text{H}}c^2 = 938.3\text{ MeV} + 0.5\text{ MeV} = 938.8\text{ MeV}.$$

The relationship between the *atomic mass unit* (u) and MeV/c^2 is

$$1\text{ u} = \frac{938.8\text{ MeV/c}^2}{1.0078} = 931.5\text{ MeV}/c^2.$$

The mass energy of the carbon atom ($A = 12$) is

$$\begin{aligned} m_{\text{C}}c^2 &= (12)(931.5\text{ MeV}) \\ &= 1.118\times 10^4\text{ MeV} = 11.18\text{ GeV}. \end{aligned}$$

■

Binding Energy

Consider an electron and proton bound together to form an atom of hydrogen. The energy (ΔE) required to separate the electron and proton to a large distance, work done against the Coulomb force, is measured to be

$$\Delta E = 13.6 \text{ eV}. \tag{1.60}$$

What happens to this energy?

The energy needed to ionize the hydrogen atom has been converted into mass energy. The mass energy of the hydrogen atom ($m_H c^2$) is smaller than the sum of the mass energies of the electron ($m_e c^2$) plus proton ($m_p c^2$) by an amount equal to 13.6 eV:

$$m_H c^2 + 13.6 \text{ eV} = m_e c^2 + m_p c^2. \tag{1.61}$$

The difference between the mass energy of the components (the electron and the proton) and the mass energy of the composite object (the hydrogen atom) is called the *binding energy* (E_b):

$$E_b = m_e c^2 + m_p c^2 - m_H c^2 = 13.6 \text{ eV}. \tag{1.62}$$

The binding energy is the amount of energy that must be provided in order to break the atom into its components. When this happens, the fractional change in mass ($\Delta m_H / m_H$) of the atom is

$$\frac{\Delta m_H}{m_H} = \frac{E_b}{m_H c^2} \approx \frac{13.6 \text{ eV}}{9.39 \times 10^8 \text{ eV}} \approx 10^{-8}. \tag{1.63}$$

For most processes, we may neglect this change in the mass of the atom.

1-6 ATOMS OF THE TWENTIETH CENTURY: QUARKS AND LEPTONS

The proton and neutron have similar masses, and both have a strong interaction of identical strength. This is a hint that they might be built of the same constituents. In 1964, Murray Gell-Mann and George Zweig independently made a classification of all known strongly interacting particles, called *hadrons*. All known hadrons could be constructed from objects that Gell-Mann called quarks after a passage in James Joyce's *Finnegans Wake*, "Three quarks for muster Mark." Three elementary building blocks were needed to make a model of the hadrons.

In 1967, Jerome Friedman, Henry Kendall, Richard Taylor, and collaborators experimentally detected the quark structure of the proton and were able to measure the momentum of the quarks inside the proton. This and other related experiments with quarks are discussed in Chapters 6, 17, and 18.

Looking Inside the Quarks

If quarks are the constituents of protons, what are the constituents of quarks? In our attempt to answer this question, we have reached the current limit of experimental resolution. The reason we are experimentally limited is the same reason that an atom cannot be resolved in an optical microscope; the wavelength of the probe is too large. We have probed quarks with a wavelength of about 10^{-18} m. This is our current experimental limit.

This is not a fundamental limit but rather a technical limit. We can summarize this by saying that the quarks are *pointlike* particles down to a distance of at least 10^{-18} m. The same is found to be true of the electron. The electron belongs to a class of particles, called *leptons*, that do not participate in the strong interaction. The quarks and the leptons have no detected structure. Today's atoms by the Greek definition are the quarks and leptons. Quarks and leptons are considered the building blocks of all matter because they are indivisible with present technology. Figure 1-13 shows the structure of matter in the universe as a function of distance of observation.

The Periodic Table of Fundamental Particles

The Quarks

Two additional quarks have been discovered since Gell-Mann and Zweig made the classification of the hadrons in 1964. The discovery of these quarks and the measurement of their properties have been crucial to our present understanding of particles and forces. The two additional quarks are much more massive than the original three, and they are often referred to as the *heavy quarks*. There are five known quarks. The original three quarks are named *up* (u), *down* (d), and *strange* (s). The heavy quarks are called *charm* (c) and *bottom* (b). The quarks are grouped into pairs by their physical properties. A periodic table of the quarks is shown in Figure 1-14.

Ordinary matter (i.e., protons and neutrons) is made of up and down quarks. A decade before its discovery, James Bjorken and Sheldon Glashow predicted that a fourth quark must exist in order to explain the properties of the other three quarks. They gave the name *charm* to the quark they predicted must exist, and they were correct! Notice that there is an empty entry in the position above the bottom quark. The properties of the bottom quark indicate

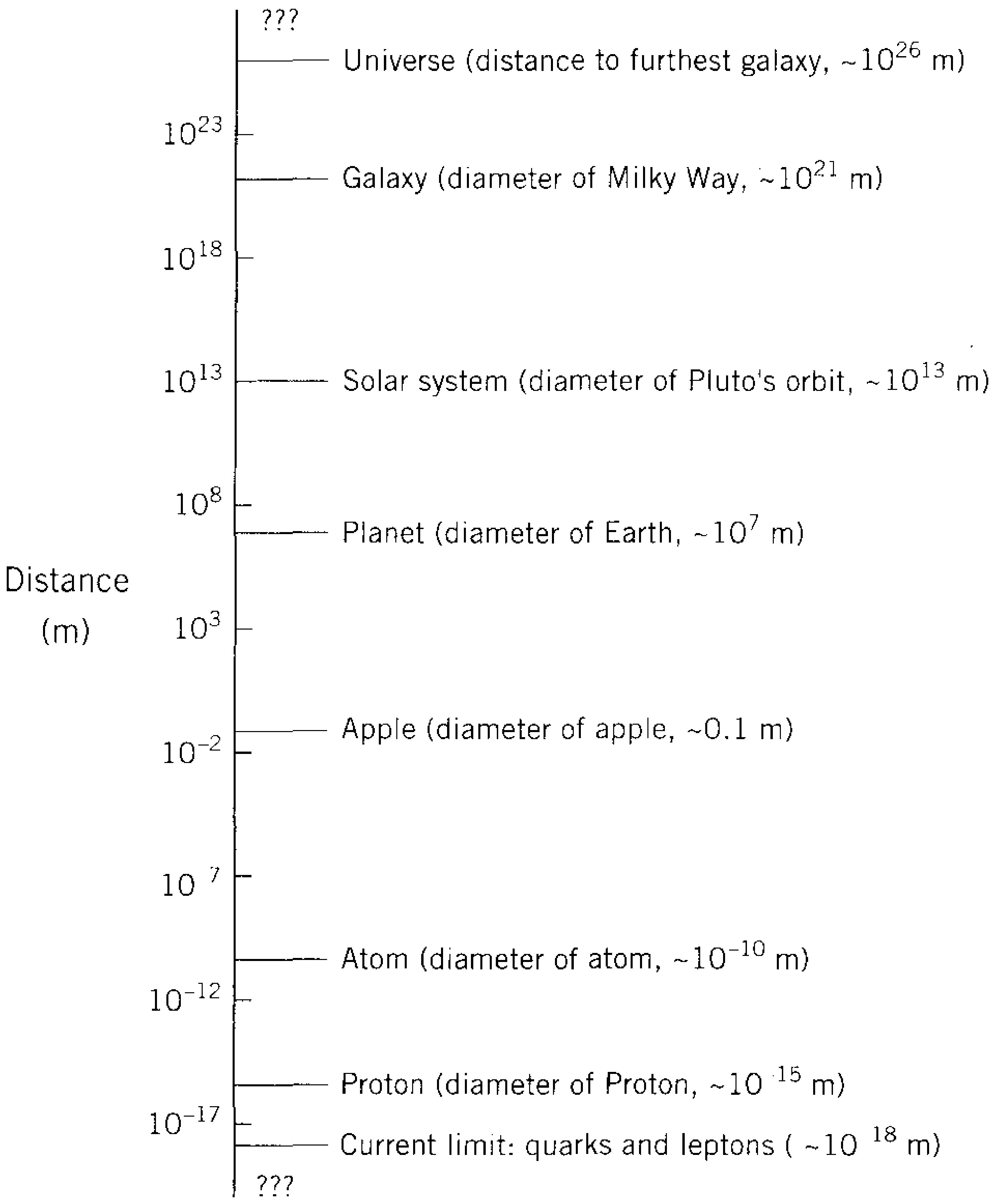


FIGURE 1-13 Sizes of various objects in our universe. The size of our universe is defined to be the distance to the furthest known object. Within our universe we find objects of varying size: galaxies, solar systems, planets, apples, atoms, protons, and finally, today's ultimate constituents: quarks and leptons.

Name charge (e)
color
symbol
mass energy

1995

Group I	Group II	Group III
Up $+\frac{2}{3}$ R, G, B *u* ~5 MeV	Charm $+\frac{2}{3}$ R, G, B *c* ~1500 MeV	Top $+\frac{2}{3}$ t ~173 GeV
Down $\frac{1}{3}$ R, G, B *d* ~10 MeV	Strange $-\frac{1}{3}$ R, G, B *s* ~150 MeV	Bottom $-\frac{1}{3}$ R, G, B *b* ~5000 MeV

FIGURE 1-14 Periodic table of the quarks. The quarks are permanently bound into hadrons. The mass energies shown are deduced from the mass energies of the particles that contain the quarks. The quarks have fractional electric charge, either +2/3 or −1/3. Each quark also has one of three types of strong charge called color: red (R), blue (B), or green (G). The blank box indicates the position expected to be occupied by a sixth quark (top).

that a sixth quark should exist. This quark has not yet been observed, because it has a mass energy beyond the sensitivity of present experiments. The sixth quark is called *top*. (Physicists sometimes refer to the bottom and top quarks by the more imaginative names of *beauty* and *truth*.)

The Leptons

In addition to the quarks, there is another group of fundamental particles called the *leptons*. Leptons are particles that do not interact strongly. The electron is the charter member of this group. The partner of the electron was postulated to exist by Wolfgang Pauli in 1931 in order to explain an apparent nonconservation of energy in decays that produce beta particles (electrons). In the 1930s, Enrico Fermi developed the theory of these decays and gave the name *neutrino* ("little neutral one") to the hypothetical particle. Finally, in 1956, the first direct observation of a neutrino particle interacting with matter was made by Frederick Reines and Clyde Cowan.

A second charged lepton, analogous to the electron, was discovered in cosmic ray experiments in 1937. The discovery of the *muon* caused a tremendous confusion for physicists, because it had no known role in nature. The discovery prompted the physicist Isidor I. Rabi to make the famous remark, "Who ordered that?" In 1961, the partner of the muon, the *muon neutrino*, was demonstrated to exist and have different properties than the partner of the electron, the *electron neutrino*. Finally, in 1975, a third *heavy* lepton was discovered. The complete family of leptons is shown in Figure 1-15.

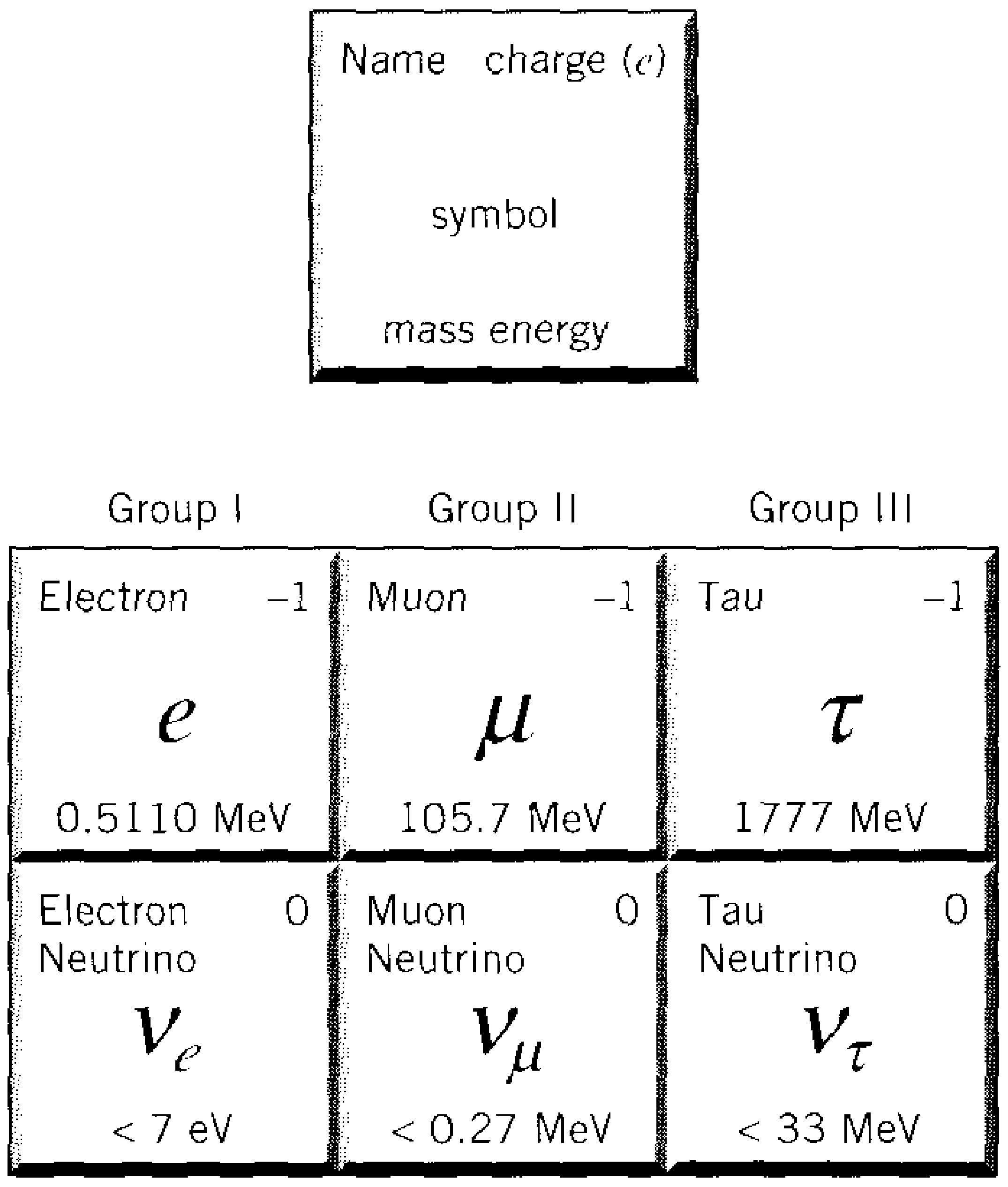
A summary of the important experiments revealing our present understanding of the structure of matter is given in Table 1-4.

* ***Challenging***

1-7 PROPERTIES OF THE FOUR FORCES

Quantum Nature of the Electromagnetic Force

The discovery of the Lorentz force law and the four Maxwell equations relating electric and magnetic fields was undoubtedly the greatest intellectual achievement



Name charge (e) symbol mass energy

Group I	Group II	Group III
Electron −1 e 0.5110 MeV	Muon −1 μ 105.7 MeV	Tau −1 τ 1777 MeV
Electron Neutrino 0 ν_e < 7 eV	Muon Neutrino 0 ν_μ < 0.27 MeV	Tau Neutrino 0 ν_τ < 33 MeV

FIGURE 1-15 Periodic table of the leptons.

of the nineteenth century. The Maxwell equations predict that charges will radiate electromagnetic energy when accelerated. This is confirmed by experiment.

Amazingly enough, a fundamental characteristic of electromagnetism remained to be discovered in the twentieth century. This additional physics is not contained in the Maxwell equations. The new phenomenon is the quantization of the electromagnetic wave! This appears in much the same manner as the quantization of electric charge. The quantization of electromagnetic radiation was first deduced by Einstein in 1905 (the same year he published the theories of Brownian motion and special relativity).

A single electromagnetic quantum is called a *photon*. Photons have nonzero energy and momentum but zero mass energy, and always travel at the speed c (in vacuum). An electromagnetic wave of a given frequency contains individual photons, each with the same energy proportional to the frequency. The experimental proof of this is discussed in Chapter 3.

TABLE 1-4
FAMOUS EXPERIMENTAL DISCOVERIES OF MODERN PHYSICS ON THE STRUCTURE OF MATTER.

Year	Experimenters	Discovery
1895	Röntgen	x rays
1896	Becquerel	α and β particles
1897	Thomson	electron
1909	Rutherford	nucleus
1932	Anderson	antimatter (positron)
1932	Chadwick	neutron
1937	Street, Stevenson, Anderson, Neddermeyer	second lepton (muon)
1956	Reines, Cowan	neutrino
1967	Friedman, Kendall, Taylor et al.	quark structure of proton
1974	Richter, Ting et al.	heavy quark (charm)
1975	Perl et al.	heavy lepton (tau)
1977	Lederman et al.	heavy quark (bottom)
1983	Rubbia et al.	W and Z^0 particles
1995		heavy quark (top)

Feynman Diagrams

When two electrons collide elastically,

$$e + e \rightarrow e + e, \qquad (1.64)$$

the momentum of each electron changes in a manner such that the total momentum of the two electrons is unchanged. We may consider the scattering as a two-step process in which an intermediate particle (γ^*) is emitted by one electron and absorbed by the other:

$$e \rightarrow e + \gamma^* \quad \text{and} \quad \gamma^* + e \rightarrow e. \qquad (1.65)$$

The intermediate particle causes the momentum transfer between the electrons. Since the intermediate particle is not free, it is called a *virtual* particle.

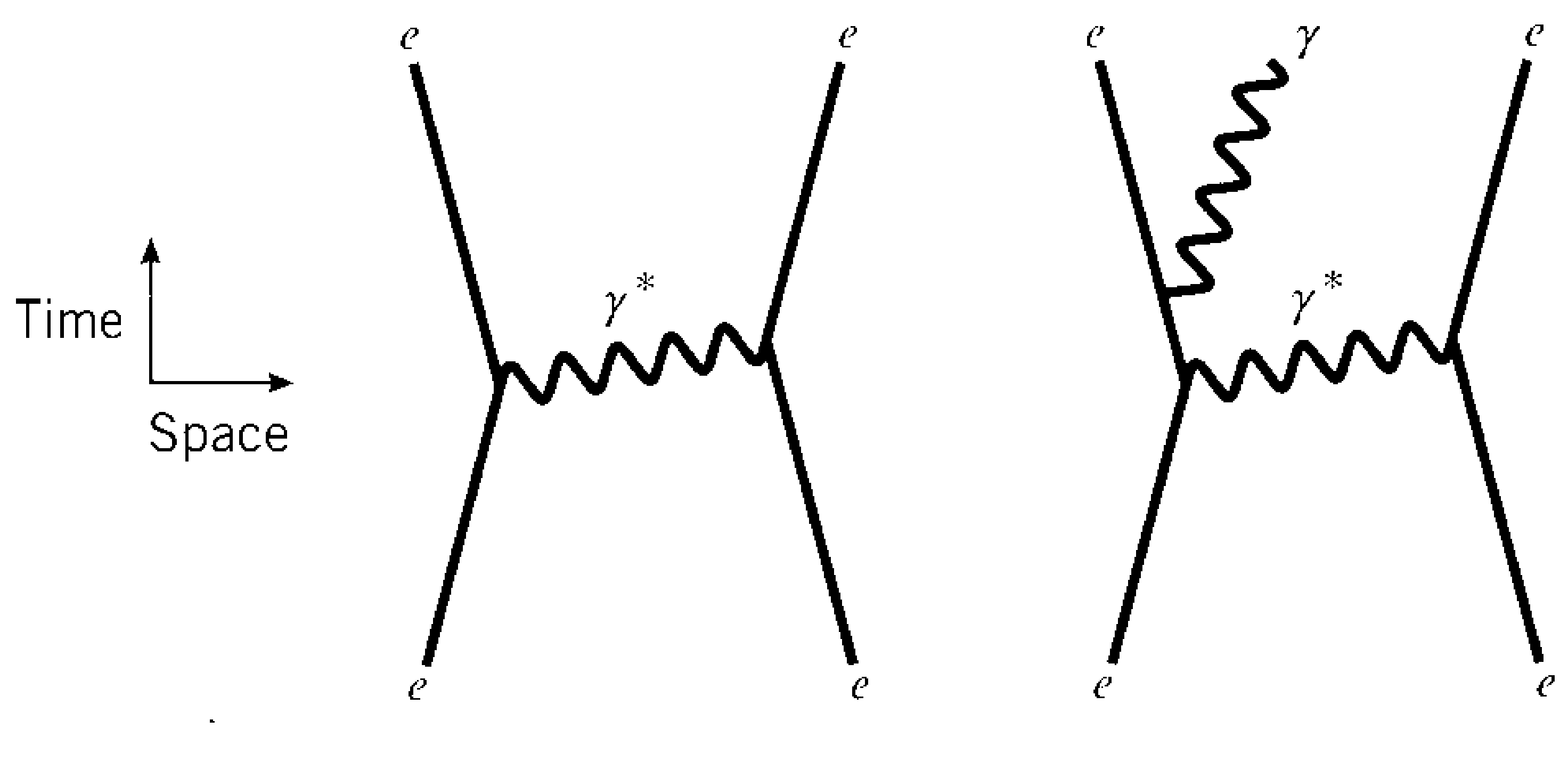
The virtual particle can cause a positive or negative momentum transfer corresponding to whether the two charges are the same or opposite in sign. The virtual particle that transmits the electromagnetic force is the photon! This is the physical mechanism by which one charge is attracted or repelled by a second charge. The two charges exchange photons, causing the transfer of energy and momentum. The theory that describes the interaction of charged particles by photon exchange is called *quantum electrodynamics* (QED). Quantum electrodynamics is the most accurate and successful theory ever constructed. The major advances in this theory were made in the late 1940s by Richard Feynman, Julian Schwinger, Freeman Dyson, and Sin-itiro Tomonaga. The theory is tested in certain cases to 10 decimal places! Nobody has ever observed a violation of any prediction of QED.

A handy pictorial representation of particle interactions was invented by Feynman, called the *Feynman diagram.* A Feynman diagram is a space–time picture for one possible path by which an interaction may occur. In the Feynman diagram, time moves upward, charged particles are represented by solid lines, and photons are represented by wavy lines. Figure 1-16*a* shows a Feynman diagram representing the scattering of two electrons. The picture presented in Figure 1-16*a* represents only one possible path, or *amplitude* for the process. The amplitude for the coupling of a photon to an electron is proportional to the charge of the electron. Photons couple to electric charge. To get the total interaction probability, we must add up all the amplitudes and then square the result. There can be interference between the different paths, analogous to the interference phenomena observed in waves. (And why not, since the photon is a wave!) Often, however, the amplitude is dominated by one or perhaps a small number of paths.

The Feynman diagrams provide a useful representation of the physical process. Figure 1-16*b* shows radiation from an accelerated electron:

$$e + e \rightarrow e + e + \gamma . \qquad (1.66)$$

In this process, called *bremsstrahlung*, a virtual photon causes an electron to be accelerated and a real photon is radiated.



(a) $e + e \rightarrow e + e$ (b) $e + e \rightarrow e + e + \gamma$

FIGURE 1-16 Feynman diagrams for electron scattering. These diagrams give a pictorial representation of the interaction of two electrons. The pictures are read from bottom to top. The solid lines represent electrons and the wavy lines represent photons. (a) Two electrons interact by the electromagnetic force through the exchange of a virtual photon (γ^*). The photon is not a free particle but is emitted by one electron and absorbed by the other. The coupling strength of the photon to the electron is proportional to the electric charge of the electron. (b) In the interaction of two electrons, a real photon (γ) is radiated.

The Range of the Electromagnetic Force

The electromagnetic force has an infinite range. This is a direct consequence of the role that photons play in mediating the force and the fact that the photon has zero mass. No matter how far apart two charges are, there is a nonvanishing force between them. The charged particles in a distant star exert a force on the electrons in your eyes, causing them to move, and this is how you can see the star even though it is a long distance away!

Introduction to Alpha

We have observed that the strength of the electromagnetic interaction (ke^2) has units of distance times energy. If we choose benchmark distance (R_0) and energy (E_0) scales, then we can represent the strength of the electromagnetic force as a dimensionless number:

$$\text{dimensionless strength} = \frac{ke^2}{R_0 E_0}. \qquad (1.67)$$

The choice of the distance and energy scales is a matter of definition but there is one and only one scale for the product R_0E_0 that occurs *naturally* in the interaction of radiation and matter. The natural scale is determined by the energy and wavelength of radiation quanta that are exchanged in the electromagnetic interaction. We may imagine that shorter wavelength radiation can get closer to the pointlike electron and in the process impart a greater momentum transfer to the electron. The energy of an electromagnetic quantum (E_{photon}) is inversely proportional to its wavelength (λ_{photon}), so that the product is a universal constant:

$$E_{\text{photon}} \lambda_{\text{photon}} = \text{constant}. \qquad (1.68)$$

This relationship gives us a natural energy and distance to evaluate the strength of the force. We shall spend all of Chapter 3 discussing the discovery and the physics of this fundamental constant. We define alpha (α) as

$$\alpha \equiv \frac{2\pi ke^2}{E_{\text{photon}} \lambda_{\text{photon}}}. \qquad (1.69)$$

The factor of 2π is arbitrarily included in the definition of α because it occurs so often in calculations. The energy of a photon that has a wavelength of 1 nm is determined from experiment to be 1240 eV. The numerical value of α is

$$\alpha \equiv \frac{2\pi(1.44\ \text{eV}\cdot\text{nm})}{(1240\ \text{eV})(1\ \text{nm})} = \frac{1}{137}. \qquad (1.70)$$

Alpha is called the *dimensionless electromagnetic coupling strength.*

We said that the natural choice of R_0E_0 was unique. One also might try to construct an energy and distance scale from the electron itself. The choice of energy scale is easy; it is the mass energy of the electron, mc^2. You might think that the distance scale could be the size of the electron. In the same manner that a photon has no definite size because its wavelength depends on its energy, the electron also has no definite size. A higher energy electron appears smaller than a lower energy electron. Furthermore, an electron always has some kinetic energy because there are always photons bumping into it. At the shortest measurable distances, the electron appears like a pointlike particle. It turns out that the electron does have a characteristic size that is determined by its energy, but the product R_0E_0 leads us back to the same constant! This wonderful and fundamental physics is the subject of Chapter 5.

Source of the Forces

Four distinct forces are observed in the interaction between various types of particles: electromagnetic, strong, weak, and gravitational. Each force is caused by some intrinsic property of the particles, analogous to electric charge.

The Strong Force

The existence of the strong force was realized with the discovery that the nuclei of atoms are bound states of neutrons and protons. The mechanism of the strong force was understood only after it was discovered that the neutrons and protons are not fundamental constituents, but are made up of quarks. The quarks have an intrinsic property called strong charge or *color*. There are *three* types of strong charges: red, green, and blue. The proton and neutron have no net strong charge, analogous to an atom having no net electric charge. The proton and neutron attract each other by the strong force in much the same manner as two atoms attract each other by the electromagnetic force to form a molecule. (The physics of how two neutral atoms can attract each other is itself an interesting question that is taken up in Chapter 10.) The quantum theory of the strong interaction is called *quantum chromodynamics* (QCD). The force between two quarks is transmitted by massless particles called *gluons*. The interaction between two quarks is qualitatively described by the Feynman diagram shown in Figure 1-17. Gluons couple to color.

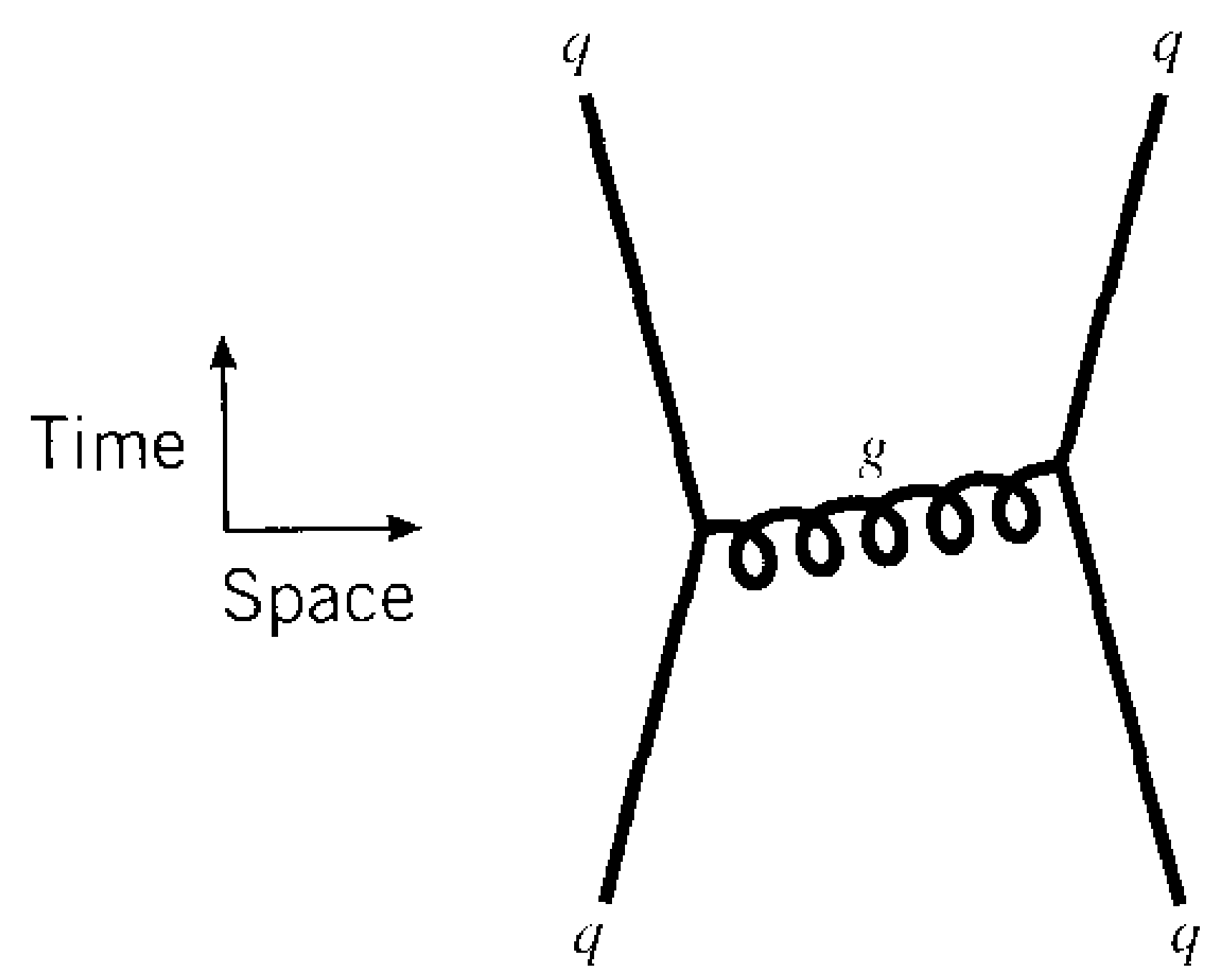


FIGURE 1-17 Feynman diagram for the strong interaction of two quarks.
The quarks (q) interact by the exchange of gluons (g).

The Weak Force

The existence of the weak force was realized with understanding of how the beta particles (electrons) were produced in the decays of nuclei. The weak force is capable of transforming neutrons into protons and vice versa. The transformation occurs because the weak interaction can cause changes in the quark flavors. The weak interaction provides the first step of the reaction chain in which protons are combined into alpha particles inside the sun and other stars. The mass energy that is released makes the sun shine!

The weak force may be described by assigning a property called *weak charge* to all quarks and leptons. The weak force between any combination of quarks and leptons is transmitted by massive particles called W^+, W^- and Z^0. The interaction between a quark and a lepton is qualitatively described by the Feynman diagram shown in Figure 1-18. The W and Z^0 particles couple to weak charge.

Gravity

The first theory of gravity was formulated by Isaac Newton and published in 1683. In spite of the fantastic

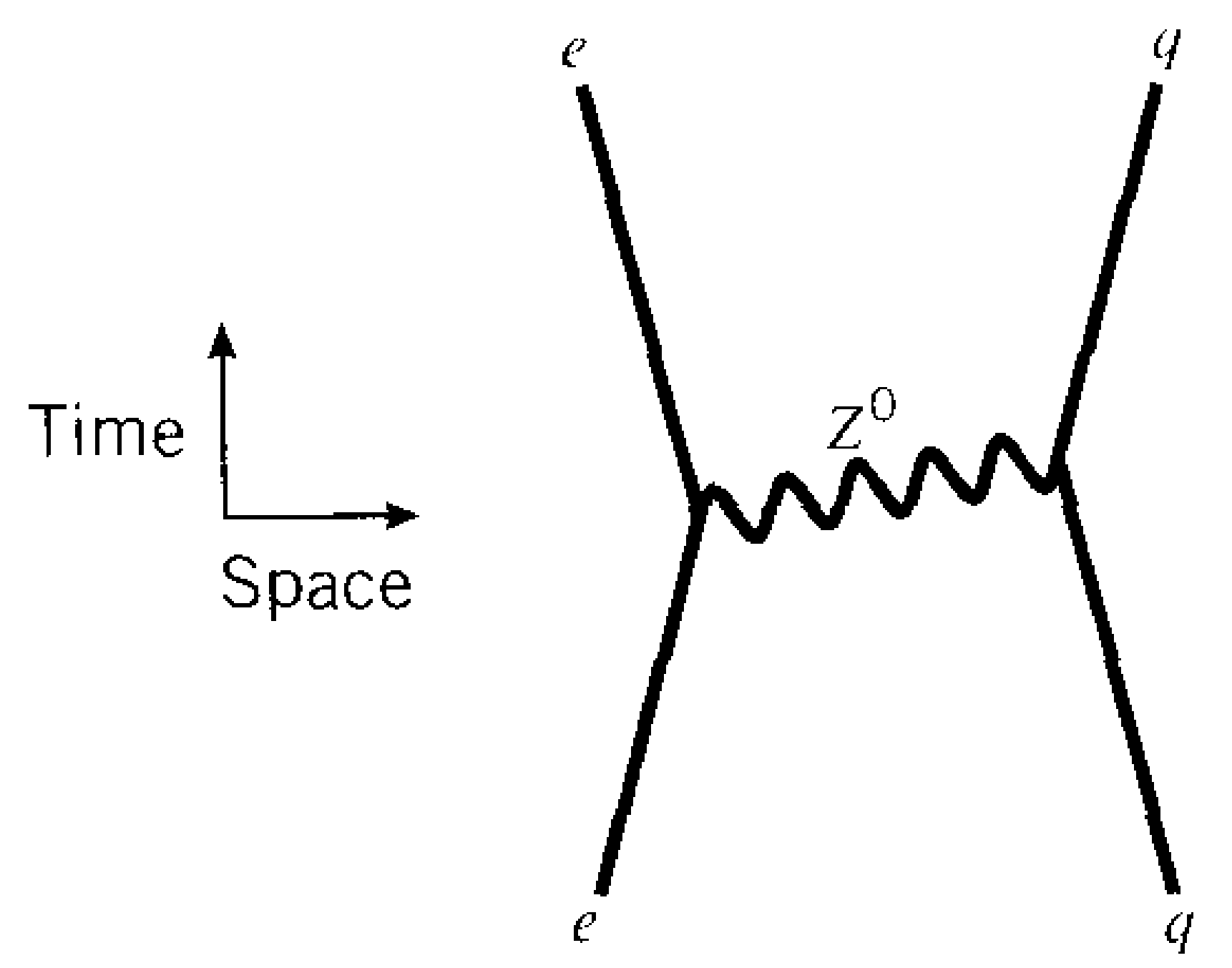


FIGURE 1-18 Feynman diagram for the weak interaction of a quark and a lepton.
The quark and lepton interact by the exchange of Z^0 particles.

success of the gravitational theory, Newton was not able to address the question of what causes the force. We now know that gravity is caused by energy. Energy is our "gravitational charge"! Recall that mass is one form of energy. It is perhaps ironic that gravity is the least understood of the four forces. There is no accepted quantum theory of gravity. The hypothetical particle that would be responsible for transmitting the gravitational force is called the *graviton*. The graviton is expected to be massless and to couple to energy, but unlike the photon, the W and Z particles, and the gluon, the graviton has not been observed!

Relative Strengths of the Forces

The four forces are equally fundamental. Our universe could not exist if any one of them were absent. Amazingly, however, they have strengths that vary by many orders of magnitude. The force between two particles depends on the particle type and also on the distance between the particles. The strength of the force of gravity between two particles depends on the mass of the particles. The gravitational force of attraction between two masses (m_1 and m_2) separated by a distance r is given by

$$\mathbf{F}_g = \frac{Gm_1 m_2}{r^2}\mathbf{i}_r . \qquad (1.71)$$

Strictly speaking, the expression for the gravitational force (1.71) is valid only if the particles are not moving at large speeds (compared to the speed of light). The gravitational coupling (Gm_1m_2) plays the analogous role of the electromagnetic coupling (kq_1q_2). The value of Gm_1m_2, for example in the attraction of the earth to the sun, must be determined from experiment. We find that

$$G = 6.67\times10^{-11}\ \mathrm{m}^3\cdot\mathrm{kg}^{-1}\cdot\mathrm{s}^{-2}. \qquad (1.72)$$

EXAMPLE 1-12

Calculate the gravitational force constant Gm_1m_2 between an electron and a proton.

SOLUTION:

The electron mass is

$$m_e = 0.511\ \mathrm{MeV}/c^2 = 9.11\times10^{-31}\ \mathrm{kg},$$

and the proton mass is

$$m_p = 938\ \mathrm{MeV}/c^2 = 1.67\times10^{-27}\ \mathrm{kg}\cdot$$

We shall leave one of the masses in kilograms and one in MeV/c^2 to arrive at units of eV·nm:

$$\begin{aligned} Gm_e m_p &= \left(6.67\times10^{-11}\ \mathrm{m}^3\cdot\mathrm{kg}^{-1}\cdot\mathrm{s}^{-2}\right)\\ &\quad\times\left(9.11\times10^{-31}\ \mathrm{kg}\right)\left(938\ \mathrm{MeV}/c^2\right)\\ &\quad\times\left(\frac{c}{3.00\times10^8\ \mathrm{m/s}}\right)\\ &= 6.33\times10^{-55}\ \mathrm{MeV}\cdot\mathrm{m}\\ &= 6.33\times10^{-40}\ \mathrm{eV}\cdot\mathrm{nm}. \end{aligned}$$

■

The ratio of strengths (gravitational to electromagnetic) between an electron and proton is

$$\frac{Gm_e m_p}{ke^2} = \frac{6.33\times10^{-40}\ \mathrm{eV}\cdot\mathrm{nm}}{1.44\ \mathrm{eV}\cdot\mathrm{nm}} \approx 4.4\times10^{-40}. \qquad (1.73)$$

The gravitational attraction between an electron and a proton is about 40 orders of magnitude weaker than the electromagnetic force. Gravity is so weak that if it was the force that governed the atomic size, then the hydrogen atom would be larger than the distance to the furthest galaxy (see Figure 1-13)!

The gravitational constant Gm_1m_2 depends on the masses of the particles. For two protons, the gravitational force constant is

$$\begin{aligned} Gm_p m_p &= \left(Gm_e m_p\right)\left(\frac{m_p}{m_e}\right)\\ &\approx \left(6.33\times10^{-40}\ \mathrm{eV}\cdot\mathrm{nm}\right)\left(\frac{938\ \mathrm{MeV}}{0.511\ \mathrm{MeV}}\right)\\ &\approx 1.2\times10^{-36}\ \mathrm{eV}\cdot\mathrm{nm}\ . \end{aligned} \qquad (1.74)$$

Recall how we specified the strength of the electromagnetic force as a dimensionless constant α by dividing by the fundamental constant (1240 eV·nm). If we do the same thing for the gravitational force between two protons, we can define the coupling strength "alpha-g" (α_g):

$$\begin{aligned} \alpha_g &\equiv \frac{Gm_p m_p}{1240\ \mathrm{eV}\cdot\mathrm{nm}}\\ &\approx \frac{1.2\times10^{-36}\ \mathrm{eV}\cdot\mathrm{nm}}{1240\ \mathrm{eV}\cdot\mathrm{nm}} \approx 10^{-39}\ . \end{aligned} \qquad (1.75)$$

The numerical value of α_g depends on the choice of mass or equivalent energy scale (m_pc^2). The above value of α_g corresponds to an energy scale of about one GeV.

Besides their widely varying strengths, there is another significant difference between the forces, which is their *range*. The gravitational and electromagnetic forces have infinite ranges, but the strong force and the weak force are observed to have extremely short ranges. The strong force binds protons and neutrons together in a nucleus, but the force between the nuclei of two neighboring atoms is zero. The reason for this is that the proton and neutron have no net strong charge, analogous to an atom having no net electric charge.

The weak force also has an extremely short range, but for a completely different reason than the strong force. The reason that the weak force has a short range is that the weak force is mediated by the exchange of very massive particles called the W and Z^0 bosons. The large masses of the W and Z^0 particles results in a short range for the weak force. The range of the weak force is given by the fundamental energy–distance constant $(1240\ \text{eV}\cdot\text{nm})$ divided by the mass energy of the W and Z^0 particles. The mass energy of these particles is approximately one hundred times the proton mass energy, or roughly 100 GeV/c^2. The range of the weak force is

$$R \approx \frac{1240\ \text{eV}\cdot\text{nm}}{10^{11}\ \text{eV}} \approx 10^{-2}\ \text{fm}. \qquad (1.76)$$

The range of the weak force is so small that the weak force is referred to as a contact interaction.

The strong force and the weak force have a much different distance dependence than the electromagnetic and gravitational forces. We can still measure the strengths of these forces and then divide by our benchmark energy–distance scale of $(1240\ \text{eV}\cdot\text{nm})$ to define the coupling strengths "alpha-s" (α_s) for the strong force and "alpha-w" (α_w) for the weak force. We have seen that the numerical value of α_g depends on the choice of energy scale. The numerical values of α_s and α_w also depend on the energy scale, *but for different reasons than α_g does.* (The strong and weak forces are the subject of Chapters 17 and 18.) The strong force between two quarks is clearly much greater than the electromagnetic force between a quark and an electron; the experimental evidence for this is that the proton size is about 10^{-15} m and the atomic size is 10^{-10} m. At an energy scale of about one GeV, the size of the dimensionless strong coupling is

$$\alpha_s \approx 1, \qquad (1.77)$$

compared to $\alpha = 1/137$. At the same energy scale of one GeV, the size of the weak coupling is

$$\alpha_w \approx 10^{-6}. \qquad (1.78)$$

The approximate relative strengths of the four forces is summarized in Table 1-5.

Gravity is far, far weaker than the other three forces. Gravity is detectable to us only because there are a large number of protons and neutrons in the earth all pulling in the same direction on us. At distances larger than a nuclear size, the strong and weak forces have zero strength and the electromagnetic force rules! The strengths and ranges of the forces are shown in Figure 1-19.

Time Constants of the Forces

Associated with each force is a time constant for the interaction. Each force is always at work to make every particle decay into a lower energy state. Of course, some particles like the electron and proton never decay, and appropriate conservation laws are invoked to explain this. It may be that the proton or the electron does decay and we have not observed it because the decay lifetime is very long!

Most particles, however, are observed to decay. A stronger force makes the decay happen faster. Often a conservation law can prevent the action of one or more forces. The decay rate is proportional to the square of the relative strength of the force (one power of alpha for each of the two particles that are coupled). The average lifetime of a particle is inversely proportional to the decay rate and therefore inversely proportional to alpha squared. The typical lifetime of a particle that decays by the strong interaction (τ_{strong}) is characterized by the time that it takes a particle traveling at the speed of light to

TABLE 1-5
COMPARISON OF THE FOUR FUNDAMENTAL FORCES.

Force	Source	Mediator	Rel. Strength (1 GeV)
Strong	strong charge	gluon	$\alpha_s \approx 1$
EM	electric charge	photon	$\alpha = 1/137$
Weak	weak charge	W and Z^0	$\alpha_w \approx 10^{6}$
Gravity	energy	graviton ?	$\alpha_g \approx 10^{-39}$

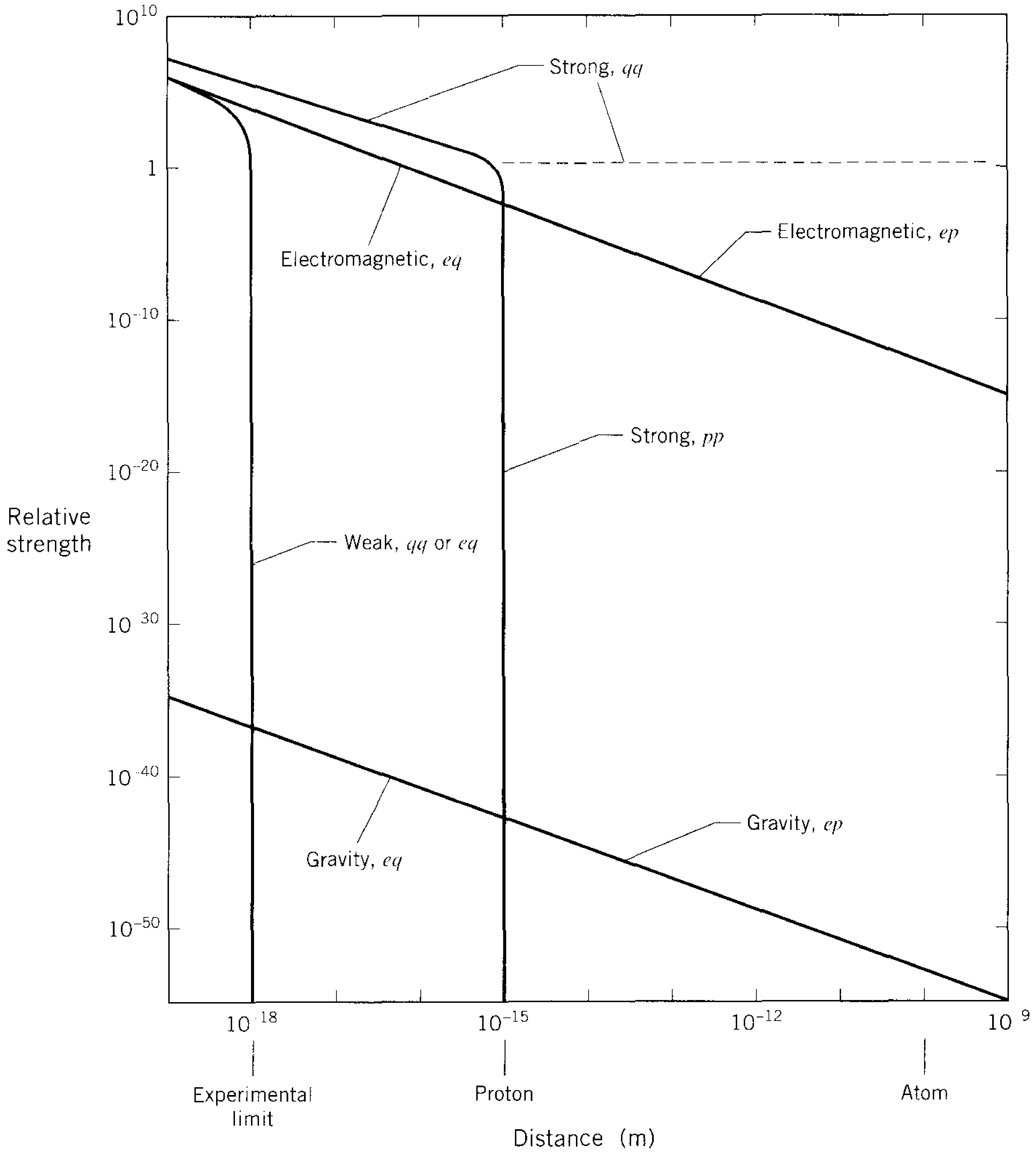


FIGURE 1-19 Strength of the four forces as a function of distance.
At atomic distances there are only two forces, electromagnetism and gravity, and the electromagnetic force between a proton and electron is about 40 orders of magnitude greater than gravity. The force between two quarks, if they could be separated, would be enormous, as shown by the dashed line. At a distance equal to the proton size, the strong force turns on abruptly to a strength about 100 times the electromagnetic force. (The strong force does not affect the electron at all.) At a distance equal to about 1/1000 of the proton size, the weak force turns on abruptly. This is the limit of current experimentation.

travel a distance equal to the size of a proton,

$$\tau_{\text{strong}} \sim \frac{\left(10^{-15}\ \text{m}\right)\alpha_s^{\ 2}}{3\times10^{8}\ \text{m/s}} \sim 10^{-23}\ \text{s}. \qquad (1.79)$$

The typical lifetime of a particle that decays by the electromagnetic interaction (τ_{em}) is characterized by a time scale of

$$\tau_{\text{em}} \sim \tau_{\text{strong}} \frac{\alpha^2{}_s}{\alpha^2} \sim 10^{-19}\ \text{s}. \qquad (1.80)$$

The typical lifetime of a particle that decays by the weak interaction (τ_{weak}) is characterized by a time scale of

$$\tau_{\text{weak}} \sim \tau_{\text{strong}} \frac{\alpha^2{}_s}{\alpha^2{}_w} \sim 10^{-11}\ \text{s}. \qquad (1.81)$$

The above order of magnitude estimates of τ_{strong}, τ_{em}, and τ_{weak} are only very crude values. The actual particle lifetimes vary widely from decay to decay because there are other factors that affect the decay rates. The main factor that sets the scale in each case, however, is "alpha" squared. The time scale of various processes in the universe are indicated in Figure 1-20.

In this introductory chapter, we have given a broad survey of particles and forces. The great majority of this text will be concerned with the physics of the electromagnetic force at the atomic distance scale (see Figure 1-19). *

CHAPTER 1: PHYSICS SUMMARY

- Matter is composed of atoms: negatively charged electrons attracted to a positively charged nucleus by the electromagnetic force. The characteristic diameter of an atom is about 0.3 nm.
- The mass of an atom is concentrated in a positively charged nucleus made up of protons and neutrons held together by the strong force. The number of neutrons plus the number of protons is called the atomic mass number (A). The mass of an atom is proportional to A. The characteristic size of a nucleus is about 1 fm.

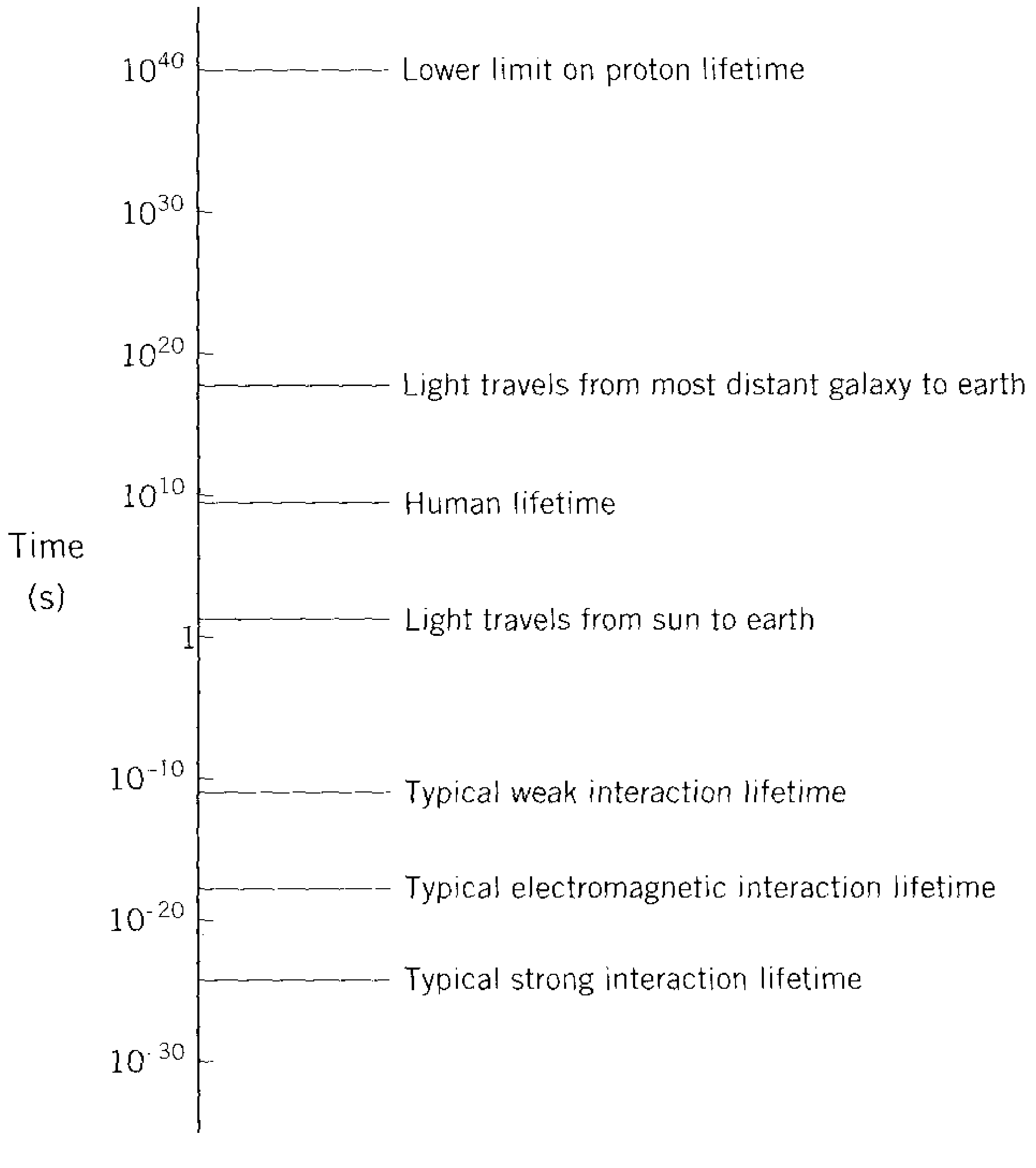


FIGURE 1-20 Time scales of various processes in the universe.

- The number of electrons or protons in an atom is called the atomic number (Z) of the element. The chemical and physical properties of the element are determined by the number of electrons in the atom. A given element may have slightly different numbers of neutrons in the nucleus. Two versions of the same element with different numbers of neutrons are called isotopes.
- Avogadro's number is defined to be the number of atoms in 12 grams of carbon with $A = 12$:

$$N_A = 6.02 \times 10^{23}.$$

For any element there are N_A atoms in A grams, where A is the atomic mass number. For any compound there are N_A molecules in M grams, where M is the sum of the atomic mass numbers of the atoms making up one molecule.

- Electric charge is quantized. The charge observed on any free particle is an integer multiple of the fundamental charge,

$$e = 1.60 \times 10^{-19} \text{ C}.$$

The electric charge of the proton is e, the charge of the electron is $-e$, and the charge of a neutron is zero. The electric charges of the proton, electron, and neutron are intrinsic particle properties that cannot be altered.

- An electromagnetic wave with a frequency f is made up of radiation quanta called photons. These quanta each have an energy proportional to the frequency of the wave and have zero mass. The radiation quanta travel at a speed,

$$c = 3.00 \times 10^{8} \text{ m/s}.$$

- The electromagnetic force is mediated by continuous photon exchange between electric charges. The strength of the electromagnetic force is specified by the constant,

$$ke^{2} = 1.44 \text{ eV} \cdot \text{nm}.$$

In dimensionless units, obtained by dividing by the energy times the wavelength of a photon, the strength of the force is

$$\alpha = \frac{2\pi ke^2}{E_{\text{photon}}\lambda_{\text{photon}}} \approx \frac{1}{137}.$$

- There are four fundamental forces in nature: strong, electromagnetic, weak, and gravity. The approximate relative strength of the four forces (α_s, α, α_w, α_g) at an energy scale of one GeV are

$$1,\ \frac{1}{137},\ 10^{-6},\ 10^{-39}.$$

  The ranges of the electromagnetic and gravitational forces are infinite, while the ranges of the strong and weak forces are extremely short. From distances of 10^{-15} m (the nuclear size) to 10^{26} m (the distance to the furthest visible galaxy) the physics of the universe is dominated by the electromagnetic interaction. The effects of gravity are noticeable when a very large number of particles are involved.

- The mass energy of a particle is potential energy stored in the form of mass. The relationship between mass energy (E_0) and mass (m) is

$$E_0 = mc^2.$$

  where c is the speed of light. The mass energy of an electron is 0.511 MeV, and the mass energy of the proton is 938 MeV.

- At 10^{-18} m, the resolution of current experiments, matter is observed to be made up of structureless particles, quarks and leptons.
- The following units are used in modern physics:

Unit	*SI*	*Modern*	*Conversion*
Length	m	nm	1 nm = 10^{-9} m
		fm	1 fm = 10^{-15} m
Energy	J	eV	1 eV = 1.60×10^{-19} J
Charge	C	e	1 e = 1.60×10^{-19} C
Mass	kg	MeV/c^2	1 MeV/c^2 = 1.78×10^{-30} kg

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QUESTIONS AND PROBLEMS

Discovery of atoms

***1.** Think about Feynman's challenge. Can you put more scientific information in fewer words?

2. A mass m_1 of carbon combines with a mass m_2 of nitrogen to form a mass M of a compound. Show that m_1/M and m_2/M are both constants.

3. One kg of hydrogen combines with 4.5 kg of carbon. What is the ratio of number of carbon to hydrogen atoms in the hydrocarbon molecule that is formed?

4. One kg of carbon combines with oxygen to make carbon monoxide (CO), carbon dioxide (CO_2), or carbon suboxide (C_3O_2). For each of the three reactions, calculate the mass of the oxide produced.

5. Determine the mass (in kg) of the following atoms: (a) ^{4}He, (b)^{12}C, and (c) ^{208}Pb.

6. Use the density of graphite, $\rho \approx 2 \times 10^3$ kg/m^3, to make an estimate of the size of a carbon atom.

7. Use the atomic number of copper, $A = 64$, to make an estimate of the density of copper.

8. The density of air at room temperature and atmospheric pressure is about 1.2 kg/m^3. Estimate the number of molecules in one cubic meter of air.

9. Estimate the number of atoms in a pencil. (*Hint:* All atoms have approximately the same size.)

10. Make an estimate of the number of atoms in a person who has a mass of 100 kg.

11. How many molecules are in (a) 1 kg of sodium-chloride (NaCl) and (b) 1 kg of sugar ($C_{12}H_{22}O_{11}$)?

Classical electromagnetism

12. Electrons in a synchrotron are made to travel in a circular orbit at a speed very close to the speed of light (c). If the radius of the orbit is 100 m, how many electrons are needed to make a current of 1 A?

13. An electric field applied to a metal causes a certain number of mobile electrons to move with an average drift speed (v_d). (a) Show that the current per area (J) is given by

$$J = nev_d,$$

where n is the density of mobile electrons. (b) A current of 1 A flows in a copper wire, which has a radius of 1 mm. Assuming that copper has one mobile electron per atom, make an estimate the average drift speed.

14. Consider a simple model of a hydrogen atom in which an electron makes a circular orbit about a stationary proton. (a) Show that the kinetic energy of the electron ($mv^2/2$) is equal to minus 1/2 times the potential energy ($-ke^2/r$). (b) An energy of 13.6 eV is required to separate the electron and proton from an orbit of radius r to some large distance ($>>r$). What is the size of the orbit? (c) What is the speed of the electron in the orbit?

***15.** How would Maxwell's equations be modified if a magnetic monopole was discovered? (*Hint:* What is the equation for conservation of monopoles?) How would the Lorentz force law be modified?

16. If $\mathbf{E}_1$ and $\mathbf{B}_1$ satisfy Maxwell's equations and $\mathbf{E}_2$ and $\mathbf{B}_2$ satisfy Maxwell's equations, show that ($\mathbf{E}_1$+$\mathbf{E}_2$) and ($\mathbf{B}_1$+$\mathbf{B}_2$) are also solutions of Maxwell's equations.

17. Consider an electromagnetic wave where the electric field has the form

$$\mathbf{E} = E_0 \cos(kz - \omega t)\mathbf{i}_x.$$

Show that the average value of the electric field amplitude squared is $E_0^2/2$.

Looking inside the atom: electrons and a nucleus

18. (a) Why can we neglect the force of gravity in the Thomson experiment? (b) Calculate the gravitational deflection of an electron in the Thomson spectrometer (see Example 1-4).

19. (a) Thomson made measurements with three different cathode materials (aluminum, iron, and platinum). They all gave the same value of q/m. What do we learn from this? (b) Thomson made measurements with three different gases initially present in his tube, which were then evacuated (air, carbon dioxide, and hydrogen). They all gave the same value of q/m. What do we learn from this?

20. In the Thomson experiment a voltage V is applied to parallel plates 0.05 m long separated by a distance of 0.01 m. Electrons are observed to be deflected by an angle of 120 milliradians. When a magnetic field of 10^{-4} T is applied, there is no deflection. (a) Calculate the speed of the electrons. (b) Calculate the increase in kinetic energy that the electrons gain from their acceleration in the electric field.

21. An electron with a speed of 10^6 m/s moves in a uniform magnetic field of 10^{-4} T. Calculate the radius of curvature of the electron's trajectory.

22. Electrons are accelerated from rest through a potential difference of 10^4 V. The electrons are then directed into a magnet that has a uniform field of 10^{-3} T. The magnetic field direction is orthogonal to the electron velocity. Calculate the radius of curvature of the electron's trajectory inside the magnet.

23. An electron and a proton each have the same radius of curvature in a uniform magnetic field. If the electron speed is 10^6 m/s, what is the speed of the proton?

24. In the Millikan oil-droplet experiment, a relatively small metal plate can pull a droplet upward with a stronger force than the whole earth pulling it downward! Do you consider this to be convincing evidence that the electric force is many, many orders of magnitude greater in strength than gravity? Can you think of other common examples that illustrate the relative strengths of electromagnetism and gravity?

25. Suppose that we designed a detection method for electrons. Could the Millikan experiment be performed directly with electrons?

26. Estimate the number of electrons in a Millikan oil droplet.

27. In the Millikan oil-droplet experiment, the mass of a droplet is 10^{-14} kg. (a) Calculate the electric field needed to overcome the force of gravity if the droplet has the charge of one electron. (b) If the area of each parallel plate is 3×10^{-3} m^2, calculate the number of excess charges present on the surface of each plate in order to produce the electric field of (a). Make a rough comparison of the number of excess charges with the number of atoms in the plates.

28. In the Millikan oil-droplet experiment, the free-fall speed of a 10^{-14} kg droplet is observed to be 10^{-3} m/s. An electric field of 3×10^5 V/m is then switched on and the droplet is observed to rise. What are the possible values of terminal rise speeds of the droplet?

29. In the Millikan oil-droplet experiment, calculate the electric field needed to make a droplet rise at the same speed as it free-falls with the field off. Take the mass of the droplet to be 10^{-14} kg and take the charge on the droplet to be the electron charge.

Looking inside the nucleus: protons and neutrons

30. Make an estimate of the mass energy density of nuclear matter.

Mass and binding energy

31. Which does your physical intuition tell you is greater, the mass energy of a mosquito or the kinetic energy of a 747 jumbo jet at cruising speed? Estimate the order of magnitude of each.

32. The energy released in the explosion of one ton of TNT is about 4×10^9 J. (a) Where does this energy come from? (b) The ^{235}U nucleus may be broken apart by bombarding it with neutrons, a process called *nuclear fission*. In the fission process, an energy of about 200 MeV is released. Where does this energy come from? (c) What mass of ^{235}U is needed to produce the equivalent of one megaton of TNT by the fission process?

33. A beam of low-energy antiprotons is directed into a hydrogen target, causing protons and antiprotons to annihilate. Assume that in the annihilation process, most of the mass of the proton and the antiproton is converted into kinetic energy. Calculate the annihilation rate that would produce one watt.

34. The deuterium nucleus (d) is a bound state of one proton and one neutron. (a) Use the masses of the n, p, and d (see Appendix K) to calculate the nuclear binding energy of deuterium. (b) The ^{238}U nucleus is a bound state of 92 protons and 146 neutrons. Calculate the nuclear binding energy of ^{238}U.

35. When an atom of carbon combines with a molecule of oxygen to produce CO_2, an energy of 11.4 eV is released. (a) How much energy is released in the burning of 1 kg of carbon? (b) How much matter is converted to energy?

36. Four protons are combined into an alpha particle by a series of nuclear *fusion* reactions that occur in the sun. The energy released in this process ($4p \rightarrow \alpha$) is about 25 MeV. If the solar luminosity (4×10^{26} W) is dominated by proton fusion, at what rate are protons "burned" in the sun?

Atoms of the twentieth century: quarks and leptons

***37.** According to our current understanding of the universe, matter is composed of six quarks with similar properties and six leptons with similar properties. Do you believe that this is an indication that the quarks and leptons might have structure? Explain!

Properties of the four forces

38. When two billiard balls collide, they appear not to exert a force on each other until they actually touch. What force is at work in the scattering of billiard balls?

39. (a) Estimate the size of an atom if the attraction of the electron and proton were due to gravity. (b) What is the typical kinetic energy of an electron in this "gravitational atom"?

40. If a tiny fraction of the molecules in two apples had an excess charge (e), what would this fraction need to be in order for the electric force between the apples to be equal in magnitude to the gravitational force between the apples?

Additional problems

***41.** Do you think that there could be a fifth fundamental force that has not yet been observed? Explain.

***42.** Consider a 1-mm thick plate of brass. (a) Make an estimate of the number of atomic layers in the plate. Why does a beam of light not penetrate the plate? (b) A beam of energetic neutrons is directed into the plate. Make an estimate of the probability that the neutron will collide with a nucleus. What do you think could happen in such a collision?

43. *Searching for proton decay.* A huge tank containing 10,000 tons (10^7 kg) of water (H_2O) is instrumented to search for proton decay. If the proton lifetime is predicted to be 10^{32} years and the detection efficiency is 50%, how many proton decay events are expected to be observed in one year?

44. Use the strength of the electromagnetic force and the size of an atom to estimate the acceleration of an electron in the hydrogen atom. How many "g" is this? ($g = 9.8$ m/s^2)

***45.** The strong force between two protons in a nucleus is about 100 times the electromagnetic force, that is, $\alpha_s \approx 100\,\alpha$. (a) What is the approximate strength of the strong force between two protons separated by a distance of 2 fm? (b) Determine the acceleration of a proton in the nucleus. How many "g" is this? Compare your answer to the acceleration of an electron in an atom (see problem 44.)

46. Show that the gravitational binding energy (E_b) of a uniform sphere of mass M and radius R is

$$E_b = \frac{3GM^2}{5R}.$$

(*Hint*: Start with an infinitesimal mass dM and calculate the energy released as mass is brought in from infinity.)