Physics II
Lab 2 – Coffee Cooling Modeling[1]

NAME:

SECTION:

PURPOSE: This is the second part of the exercise started last time. Here, you will search for a mathematical description of the experimental data you found in the "coffee cooling" experiment.

Part II

PROCEDURE: You need to run Microsoft Excel to do this part of the exercise. Load up your experimental data from Part I. You should have the following columns:

1. Time. Make sure the first entry corresponds to $t = 0$.
2. Water temperature T.
3. Difference between water temperature and room temperature ($\Delta T = T - T_r$).
4. $y(t) = -\ln(\Delta T/\Delta T_0)$, where ΔT_0 is the value of ΔT at $t = 0$.

Now do the following:

1. Plot $y(t)$ and get Excel to produce a least-squares fit. You do this by pulling down the "Chart" menu, and going to "Add trendline". Click the "options" tab and check the boxes for "set intercept = " (set it equal to zero to force the fit to go through the origin), "display equation on chart", and display the R^2 value on chart". Record the value of the slope to 11 decimal places (you will need to double-click on the equation on the graph, and format the display accordingly). Here, the computer is doing the following: it assumes a straight line of the form $Y(t) = a + mt$ (but you forced it to set $a = 0$); it makes some guess for m, then for every value of t you have, it computes $Y(t)$ and squares the difference between $y(t)$ and $Y(t)$; then it sums all those squares; it remembers this number and makes another guess for m; then it computes the new $Y(t)$, squares the difference, and sums the squares, and so forth until the value it gets for the sum of the squares is as small as possible.

[1]Important: Read this in its entirety before doing the lab.

2. Now do the least-squares fit manually as follows: set two columns aside. You will leave most of these entries blank. In one column, enter "$m =$" ,"$b =$", and "$c =$". To the right of these cells, enter 0, 0, and 0.

3. Now pick another column and enter the following formula for the top cell and then copy the formula down the column: =(cell for m) * (cell for corresponding t). Make sure you put "$" signs appropriately so that the value of m is the same for every cell.

4. Now take the column to the right of the column above, and enter in the first cell: =((cell for Y) - (cell for y))ˆ2. Then copy this formula (click and drag) all the way down to the end of the data.

5. Pick a cell in some convenient place on the sheet, and enter =sum(cell1:cell2), where cell1 is the the first cell in the step above, and cell2 is the last one. This adds up all those numbers.

6. What you need now is to make the sum above as small as possible by changing your entry for m. Do this a few times to see the (immediate) effect on the sum of the squares.

7. Fortunately, Excel has an algorithm for guessing – so you do not have to do the step above many times before you find a satisfactory answer. Pull down the "Tools" menu and click on "Add-Ins". Check the box for "Solver Add-In". This loads the "Solver" so you can actually use it. (This mini-"Toolbox" is not automatically loaded when you bring up Excel, so you have to load it manually.) To use the solver now, you click on "Tools", scroll down to "Solver", and click on it. Enter the cell address for the sum of the squares in the box for "Target cell", check the "Min" button, and enter the cell address for m in the box labeled "By Changing". Then click on "Solve". Write down the value for m and for the sum of the squares to eleven decimal places. How does this value of m compare with what you got above from "Adding a Trendline"?

   Notice that this step minimizes the quantity s = sum of squares = $s(m)$, *i.e.* the one parameter m is changed until s is as small as possible. You are minimizing in 2 dimensions.

8. Now do steps 3-7 but with the formula $Y = m * t + b * t^2$, and get Excel to minimize the sum of the squares by changing the values of both m and b. (Notice that now $s = s(m, b)$, and you are minimizing in 3 dimensions.) Record these values, and the value of s to eleven decimal places. This part is tricky because now you have to give an initial guess for *two* parameters (m and b) instead of only one, as in step 7. It turns out that $s = s(m, b)$ has many local minima. You are looking for the global minimum. You give the solver an initial guess for m and b, and it returns close-by values for m and b which correspond to a

local minimum of $s = s(m, b)$. You will need to do this part of the procedure a few times to see how small a sum of squares you can come up with. Notice that because you have two parameters instead of only one (as in step 7), your sum of squares should be smaller than in step 7. Two pieces of advice:

(a) The new best value for m should be different from the one in step 7 (again, because you now have two parameters instead of only one). Because the graph of your experimental data should "curve down", the new best m should be slightly larger than the old one – the second order term should "push down" the fit for large values of t.

(b) If you are at a loss as to what to guess, try the following. Go back to your original graph of $Y(t)$, and do a trendline. This time, however, do a polynomial fit of $O(2)$. (Force the trendline to go through the origin.) Excel will return to you its own best guess for m and b. You can then use this or close-by values for your own guesses.

9. Now do steps 3-7 but with the formula $Y = m*t+b*t^2+c*t^3$, and get Excel to minimize $s = s(m, b, c)$ by changing the values of all m, b, and c. Record these values, and the value of the sum of the squares to eleven decimal places. (The same advice applies here as in step 7.) Notice how complicated it becomes to find the global maximum of a function in several dimensions.

10. Turn in these instructions and the following plots (use a symbol for the experimental data $y(t)$, and a thin line for the "best fit" $Y(t)$):

    (a) $y(t)$ and trend line from Excel.

    (b) $y(t)$ and your own linear fit.

    (c) $y(t)$ and your own fit of order 2.

    (d) $y(t)$ and your own fit of order 3.

11. Turn in your answers to the questions in the text above (see underlined text), and also discuss the "goodness of fit" of the various graphs you made, and whether it would be advantageous to continue computing more terms in the series $Y = m*t+b*t^2+c*t^3+d*t^4+....$ Also, write down your final best fit for $T(t)$. (Please type.)