Resistances in Series and Parallel

I. INTRODUCTION

The circuit components of simple circuits are connected in series and/or parallel arrangements. Each component may be represented as a resistance to the flow of current in the circuit. In computing the voltage and current requirements of the circuit (or part of the circuit), it is necessary to know the equivalent resistances of the series and parallel arrangements.

In this experiment, the circuit characteristics of resistors in series and parallel will be investigated. A particular circuit will first be analyzed theoretically and then those predictions will be checked experimentally.

II. EQUIPMENT NEEDED*

- Battery or power supply (3 V)
- Ammeter (0 to 500 mA)
- Voltmeter (0 to 3 V)
- Single-pole, single-throw switch
- Four resistors (10 Ω, 20 Ω, 100 Ω, and 10 kΩ, composition type, 1 W)
- Connecting wires

III. THEORY

A. *Series Resistance*

Resistors are said to be connected in **series** when they are connected as in Figure 33.1. (The resistors are connected in line or "head to tail," so to speak, although there is no distinction between the connecting ends of a resistor.) When connected to a voltage source V and the switch closed, the source supplies a current I to the circuit.

By the conservation of charge, this current I flows through each resistor. However, the voltage drop across each resistor is not equal to V, but the sum of the voltage drop is:

$$V = V_1 + V_2 + V_3 \tag{33.1}$$

In an analogous liquid-gravity circuit (Figure 33.1), a pump, corresponding to the voltage source, raises the liquid a distance h.† The liquid then falls or "drops" through three paddle-wheel "resistors" and the distances h_1, h_2, and h_3. The liquid "rise" supplied by the pump is equal to the sum of the liquid "drops," $h = h_1 + h_2 + h_3$. Analogously, the volt-

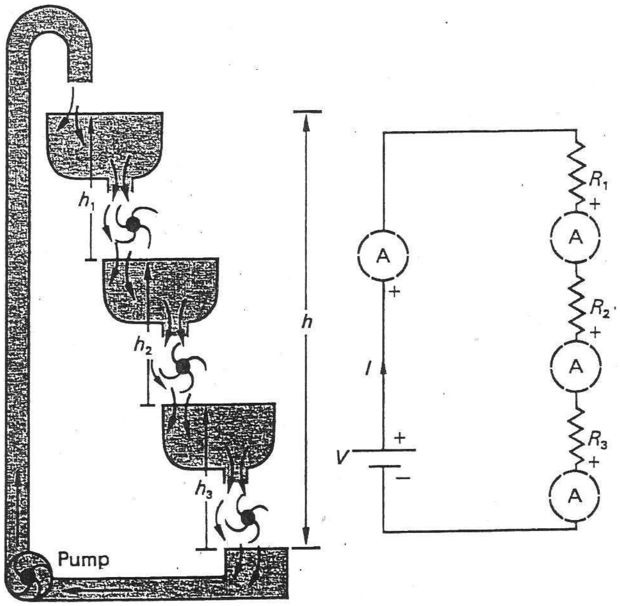

Figure 33.1 Series connection. Liquid analogy and circuit diagram for resistors in series. See the text for a description.

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*The ranges of the equipment are given as examples. These may be varied to apply to available equipment.

†Keep in mind that an analogy represents only a resemblance. Liquid and electrical circuits are quite different physically.

age "rise" supplied by the source is equal to the sum of the voltage drops across the resistors (Eq. 33.1).

The voltage drop across each resistor is given by Ohm's law (e.g., $V_1 = IR_1$). Eq. 33.1 may be written

$$\begin{aligned} V &= V_1 + V_2 + V_3 \\ &= IR_1 + IR_2 + IR_3 \\ &= I(R_1 + R_2 + R_3) \qquad (33.2) \end{aligned}$$

For a voltage across a single resistance R_s in a circuit, $V = IR_s$, and by comparison,

$$R_s = R_1 + R_2 + R_3 \qquad (33.3)$$

where R_s is the equivalent resistance of the resistors in series. That is, the three resistors in series could be replaced by a single resistor with a value of R_s and the same current I would flow from the battery.

B. *Parallel Resistance*

Resistors are said to be connected in **parallel** when connected as in Figure 33.2. (In this arrangement, all the "heads" are connected together, as are all of the "tails.") The voltage drops across all the resistors are the same and equal to the voltage V of the source. However, the current I from the source divides among the resistors such that

$$I = I_1 + I_2 + I_3 \qquad (33.4)$$

In the liquid circuit analogy (Figure 33.2), the height h the pump raises the liquid is equal to the distance the liquid "drops" through each paddle-wheel "resistor." The liquid flow coming into the junction of the parallel arrangement divides among the three pipe paths, analogously to the current dividing in the electrical circuit.

The current in an electrical parallel circuit divides according to the magnitudes of the resistances in the parallel branches—the smaller the resistance of a given branch, the greater the current through that branch. The current through each resistor is given by Ohm's law (e.g., $I_1 = V/R_1$) and Eq. 33.4 may be written

$$\begin{aligned} I = I_1 + I_2 + I_3 &= \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3} \\ &= V\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) \qquad (33.5) \end{aligned}$$

For the current through a single resistance R_p in a circuit, $I = V/R_p$, and by comparison

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \qquad (33.6)$$

where R_p is the equivalent resistance of the resistors in parallel. That is, the three resistors in parallel could be replaced by a single resistor with a value of R_p and the same current I would flow from the battery.

The previous developments for equivalent resistances may be extended to any number of resistors (i.e., $R_s = R_1 + R_2 + R_3 + R_4 + \ldots$ and $1/R_p = 1/R_1 + 1/R_2 + 1/R_3 + 1/R_4 + \ldots$).

In many instances, two resistors are connected in parallel in a circuit, and

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2}$$

or

$$R_p = \frac{R_1 R_2}{R_1 + R_2} \qquad (33.7)$$

This particular form of R_p for two resistors may be more convenient for calculations than the reciprocal form.

Also, in a circuit with three resistors in parallel, the equivalent resistance of two of the resistors can be found by Eq. 33.7, and then the equation may be applied again to the equivalent resistance and the other resistance in parallel to find the total equivalent resistance of the three parallel resistors. However, if your calculator has a $1/x$ function, the reciprocal form may be easier to use.

Note that the voltage drops across R_1 and R_2 in parallel are the same, and by Ohm's law,

$$I_1 R_1 = I_2 R_2$$

or

$$\frac{I_1}{I_2} = \frac{R_2}{R_1} \qquad (33.8)$$

Thus, the ratio of the resistances gives the relative magnitudes of the currents in the resistors.

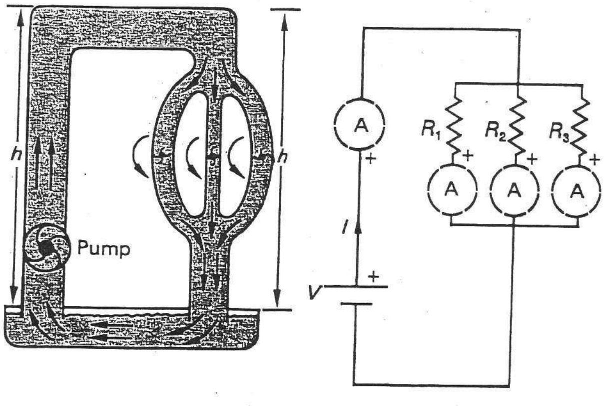

Figure 33.2 Parallel connections. Liquid analogy and circuit diagram for resistors in parallel. See the text for a description.

EXAMPLE 33.1 Given two resistors, R_1 and R_2, with $R_2 = 2R_1$ in parallel in a circuit, what fraction of the current I from the voltage source goes through each resistor?

Solution With $R_2 = 2R_1$, or $R_2/R_1 = 2$, by Eq. 33.8

$$I_1 = \left(\frac{R_2}{R_1}\right) I_2 = 2I_2$$

Since $I = I_1 + I_2$, we have

$$I = I_1 + I_2 = 2I_2 + I_2 = 3I_2$$

or

$$I_2 = \frac{I}{3}$$

Hence, the current divides with one-third going through R_2 and two-thirds going through R_1.

Consider the circuit in Figure 33.3. To find the equivalent resistance of this series-parallel circuit, one first "collapses" the parallel branch into a single equivalent resistance, which is given by Eq. 33.7. This equivalent resistance is in series with R_1 and the total equivalent resistance R of the circuit is $R = R_1 + R_p$.

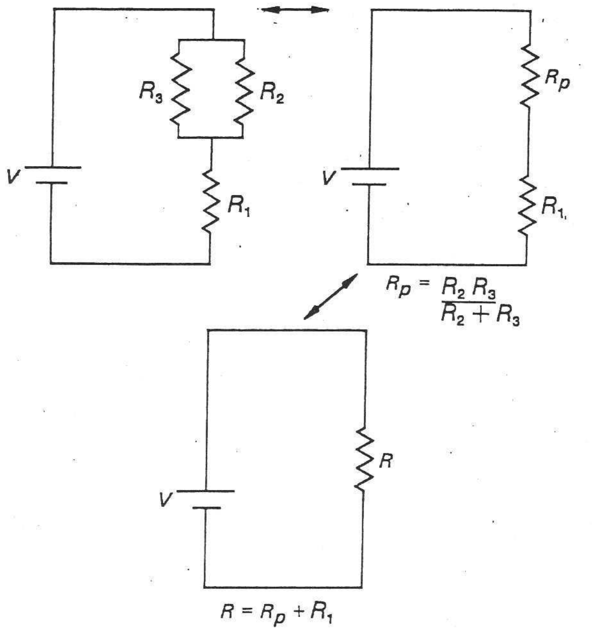

Figure 33.3 Circuit reduction. Series and parallel resistances are combined to find the equivalent resistance of a series-parallel circuit.

IV. EXPERIMENTAL PROCEDURE

1. Examine the resistors. The colored bands conform to a color code that gives the value of a resistor. Look up the color code in Appendix A, Table A5, and read the value of each resistor and record in the Laboratory Report. Designate the smallest resistance as R_1, and consecutively larger values as R_2, R_3, and R_4.

2. In the following procedures, you will be asked to compute theoretically various quantities for a given circuit arrangement. The quantities are then determined by actual circuit measurements and the calculated and experimental results compared. Before initially activating each circuit arrangement, *have the circuit checked by the instructor, unless otherwise instructed.*

A. *Resistors in Series*

3. If R_1, R_2, and R_3 are connected in a circuit in series as in Figure 33.1, (a) what is the current flowing in the circuit; (b) what is the voltage drop across each resistor? Show your calculations in the Laboratory Report. Ask your instructor for the voltage of the source if this is not known.

4. Set up the actual circuit with a switch and only one ammeter in the circuit placing it next to the current source. A convenient way to check a circuit to see if it is properly connected is to trace the path of current flow (with your finger) through the circuit. Do this for the circuit under consideration to make sure that the current will flow through each circuit component in series. Remember, an ammeter is *always* connected in series, and for proper polarity, + is connected to +.

   Close the switch (after having the circuit checked by the instructor) and read and record the ammeter value of the current (I) from the voltage source. If the needle on the ammeter goes in the wrong direction, reverse the polarity, i.e., the hookup of the leads to the ammeter. (If using a variable power supply, adjust voltage, if required, to 3.0 V with circuit closed. Otherwise, the voltage drop in the power supply may yield significantly less than 3 V to the external circuit.)

Open the switch and move the ammeter in the circuit to a position "after" a resistor [i.e., on the opposite side of the resistor from the voltage source so as to measure the current flowing through (from) the resistor]. The ammeter positions are shown in Figure 33.1. Carry out this procedure for each resistor and record the currents in the Laboratory Report. Leave the switch closed only while readings are being taken.

5. Remove the ammeter from the circuit, and with the voltmeter, measure and record the voltage drop across each resistor and across all three resistors as a group. Remember, a voltmeter is *always* connected in parallel or "across" a circuit element to measure its voltage drop.

6. Compare the experimentally measured values with the theoretically computed values by finding the percent error. (Use the theoretical values as the accepted values.)

B. *Resistors in Parallel*

7. If R_1, R_2, and R_3 are connected in a circuit in parallel as in Figure 33.2, (a) how much current (I) is supplied by the source; (b) what is the current flowing through each resistor? Show your calculations in the Laboratory Report.

8. Set up the actual circuit. Check the circuit arrangement by tracing the current from the source through the circuit to see that it divides into three parallel branches at the junction of the resistors and comes together again at the opposite junction. Then measure and record the voltage drops across each resistor and across all three resistors as a group.

   As before and throughout the remaining procedures, leave the switch closed only while readings are being taken.

9. Remove the voltmeter and connect the ammeter between the source and resistors so as to measure the current I supplied by the source. Measure and record this current. Then measure the current through each resistor by connecting the meter between a given resistor and one of the common junctions. The ammeter positions are shown in Figure 33.2.

10. Compare the theoretical and experimental values by computing the percent errors.

11. *(Optional)* Repeat procedures 7 through 10 with R_2 replaced by R_4.

C. *Resistors in Series–Parallel*

12. If R_1 is connected in series with R_2 and R_3 in parallel (Figure 33.3):
    (a) How much current is supplied by the source?
    (b) What is the voltage drop across R_1?
    (c) What is the voltage drop across R_2 and R_3?
    (d) What is the voltage drop across all three resistors?
    (e) What are the currents through R_2 and R_3?

13. Set up the actual circuit and trace the current flow to check the circuit. With the voltmeter and ammeter, measure and record the calculated quantities.

    You need not compute the percent errors in this case. However, make a mental comparison to satisfy yourself that the measured quantities agree with the computed values within experimental error.

Resistances in Series and Parallel

Laboratory Report

Resistor values R_1 ______ R_3 ______

R_2 ______ R_4 ______

A. *Resistors in Series*

Calculations
(show work)

Current I ______

Voltage drops across resistors V_1 ______

V_2 ______

V_3 ______

Experimental measurements

				Percent error
I	______			______
I_1	______	V_1	______	______
I_2	______	V_2	______	______
I_3	______	V_3	______	______
		V	______	

B. *Resistors in Parallel*

Calculations
(show work)

Current I ______

Current through resistors I_1 ______

I_2 ______

I_3 ______

Don't forget units

(continued)

Experimental measurements

Percent error

I ____________ ____________

V_1 ____________ I_1 ____________ ____________

V_2 ____________ I_2 ____________ ____________

V_3 ____________ I_3 ____________ ____________

(Optional Procedure)

Calculations
(show work)

Current I ____________

Current through resistors I_1 ____________

I_3 ____________

I_4 ____________

Experimental measurements

Percent error

I ____________ ____________

V_1 ____________ I_1 ____________ ____________

V_3 ____________ I_3 ____________ ____________

V_4 ____________ I_4 ____________ ____________

C. *Resistors in Series-Parallel*

Calculations
(show work)

Current I ________________

Voltage drops

V_1 ________________

$V_2 = V_3$ ________________

Currents

I_2 ________________

I_3 ________________

Experimental measurements

I ________________

V_1 ________________

$V_2 = V_3$ ________________

I_2 ________________

I_3 ________________

QUESTIONS

1. Discuss the sources of error in the experiment.

2. Suppose that the resistors in the various circuit diagrams represented the resistances of light bulbs in such circuits. When a light bulb "burns out," the circuit is open through that particular component, i.e., R is infinite. Would the remaining bulbs continue to burn for the following conditions? If so, would the bulbs burn more brightly (draw more current) or burn more dimly (draw less current), if
   (a) R_2 burned out in the circuit in part A?

(b) R_1 burned out in the circuit in part B?

(c) Then R_3 also burned out in the circuit in part B?

(d) R_3 burned out in the circuit in part C?

(e) Then R_1 also burned out in the circuit in part C?

3. Explain the effect of replacing R_2 with R_4 in procedure 11. (Explain theoretically even if procedure 11 of the experiment was not done.)

4. Given the four resistors in the experiment, how many possible different resistance values could be obtained by using one or more of the resistors? [List the specific combinations (e.g., R_1 and R_2 in series).]