Bose-Einstein Condensation of Excitons in Two-Dimensions

Alexander Dzyubenko

Department of Physics, California State University at Bakersfield
Department of Physics, University at Buffalo, SUNY
General Physics Institute, Moscow

Supported in part by NSF

MPIPKS, Dresden
July 20, 2004

Collaboration

P. I. Arseev (Lebedev Physical Inst., Moscow)

G. E. W. Bauer, Yu. V. Nazarov (TU Delft)

L. V. Butov (ISSP, Chernogolovka; UCSD)

Yu. E. Lozovik (Inst. of Spectroscopy, Troitsk)

Outline

v Bose-Einstein Condensation (BEC)

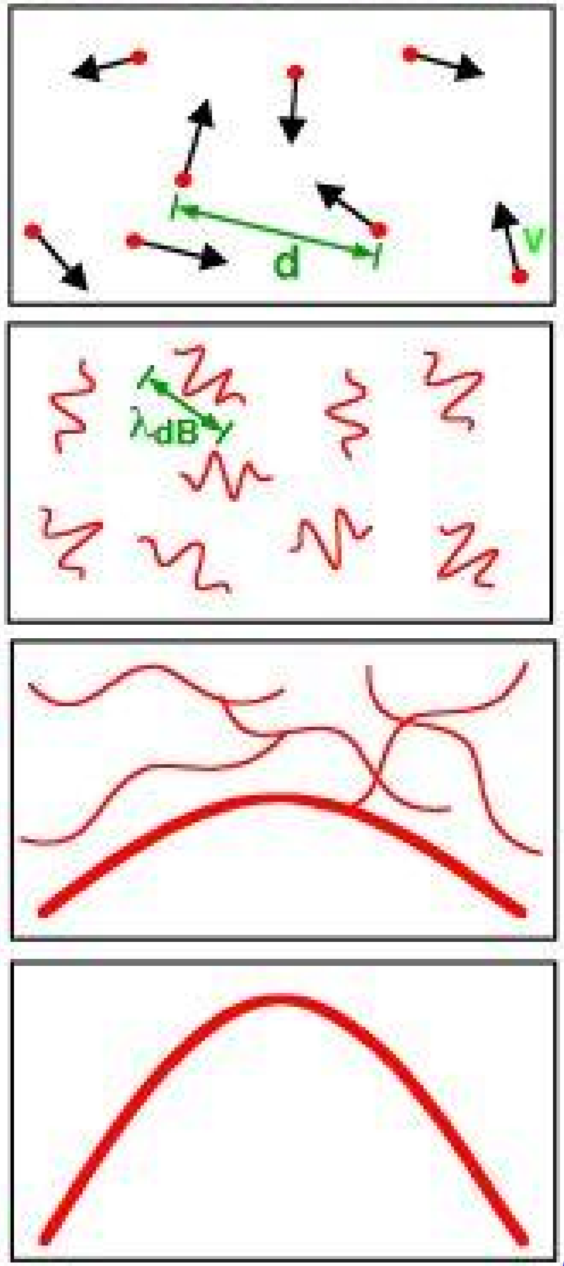
Ø BEC of composite particles: 2D magnetoexcitons

Class of exactly-solvable models

"Hidden Symmetry"

v Short Summary

What is Bose-Einstein condensation (BEC)?



High Temperature T:

thermal velocity v
density d^{-3}

"Billiard balls"

Low Temperature T:

De Broglie wavelength
$\lambda_{dB}=h/mv \propto T^{-1/2}$

"Wave packets"

$T=T_C$
Bose-Einstein Condensation

$\lambda_{dB} \approx d$

"Matter wave overlap"

T=0:
Pure Bose condensate

"Giant matter wave"

Here shown in a trap

$$\frac{4p}{3}d^3 n = 1$$

$$l_{dB} = \left(\frac{2p\mathbf{h}^2}{mk_{\mathrm{B}}T}\right)^{1/2}$$

$$T_c \propto \frac{n^{2/3}}{m}$$

http://cua.mit.edu/ketterle_group/

Bose Gas at T=0

$$[\hat{a}_{K'}, \hat{a}_K{}^{+}] = d_{K',K}$$

Non-interacting Bose Gas

$$H_0 = \sum_K E_K\, \hat{a}_K{}^{+} \hat{a}_K$$

Quantum Equation of Motion

$$[H_0, \hat{a}_K{}^{+}] = E_K\, \hat{a}_K{}^{+}$$

The Ground State: *K* = 0 Condensate

$$\left(\hat{a}_0{}^{+}\right)^N |\,0\rangle \equiv |\,N\rangle$$

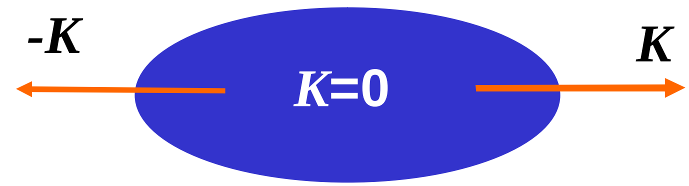
$$H_0\,|\,N\rangle = NE_0\,|\,N\rangle$$

Interacting Bose Gas

add

$$H_{\text{int}} = \sum U_{K_1 - K_1'}\, \hat{a}_{K_1}{}^{+} \hat{a}_{K_2}{}^{+} \hat{a}_{K_2'} \hat{a}_{K_1'}$$

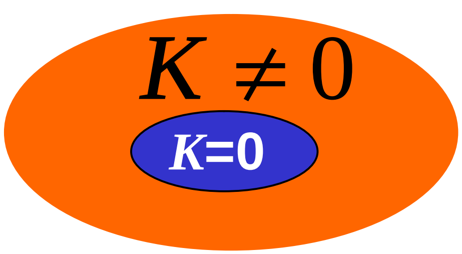
Depletion of the *K*=0 State



$$U_K\, \hat{a}_K{}^{+} \hat{a}_{-K}{}^{+} \hat{a}_0 \hat{a}_0$$

Ground State:

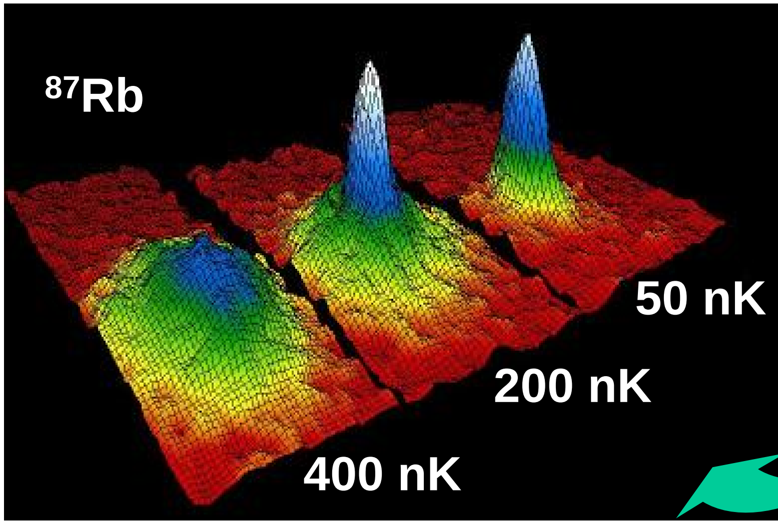
Condensate formed $|\,\Phi\rangle$



^{4}He: ***K* = 0 fraction ~ 0.1** **(low T)**

BEC in Dilute Ultra-Cold Atomic Gases

^{87}Rb, ^{23}Na, ^{7}Li, ^{41}K, ^{1}H, $^{4}He^*$



Sharp peak in the *velocity distribution*

# of condensed atoms reached: ~ 10^6

2001 Nobel Prize in Physics
Cornell, Wieman & Ketterle

Densities **$n \sim 10^{14}$ cm^{-3}**
(~10^{-5} times the density of air)

Temperatures **($T_c \sim 170$ nK for ^{87}Rb)**
Evaporative cooling to ~ 20 nK
in magneto-optical traps

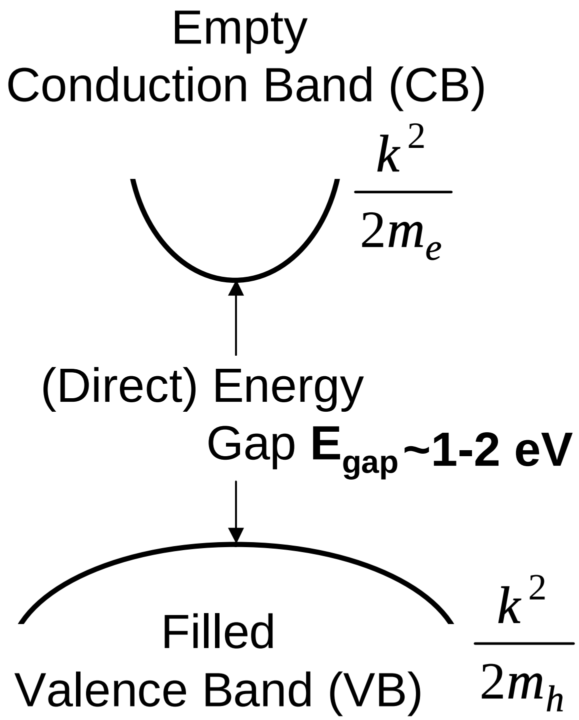
BEC is *single quantum object*
(size ~ 50 μm)

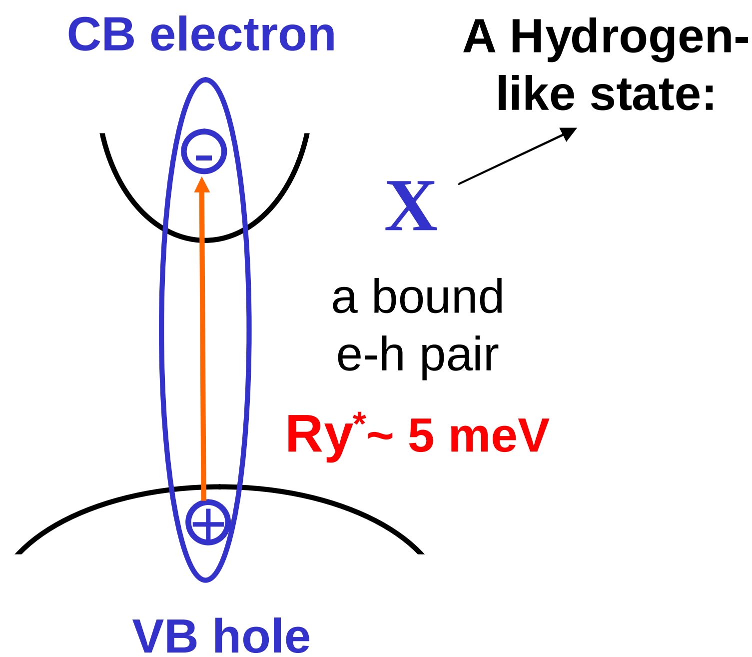
- **BEC superfluidity**

- **BEC of ^{7}Li (Boson)**

+ ^{6}Li *(Fermion) gas*

Excitons (X) in Semiconductors





Effective Mass Approximation:

$m_e^* \sim 10^{-2}\, m_e$ $m_h^* \sim 10^{-1}\, m_e$ $e \sim 10$ **Large radius (Wannier-Mott) excitons**

$1/m^* = 1/m_e^* + 1/m_h^*$ **X reduced mass**

$a_B^* = e h^2 / m^* e^2 \sim 10^2$ Å **>> lattice constant ~ 3** Å

$Ry^* = e^4 m^* / 2 e^2 h^2$

BEC of Excitons

- a **light**, mobile, neutral particle $m^* \sim \frac{m_{\text{proton}}}{2000} \times \frac{1}{10}$
- consists of two Fermions: **a Boson**

X $\Leftrightarrow$ $T_c = 3.31 \frac{\hbar^2 n^{2/3}}{m^* k_B}$

$$k_B T_c = 3.31 \frac{\hbar^2}{m^* (a_B^*)^2} [n(a_B^*)^3]^{2/3} \sim Ry^*$$

high density $n(a_B^*)^3 \sim 1$

$T_c \sim 50\,\text{K}$

$n \sim 10^{18}\,\text{cm}^{-3}$

Blatt *et al.* 1962 **Moskalenko 1962**

§ finite **X** radiative lifetimes (wide range! ~ ps – ms) $\Longrightarrow$ Optical excitation: high n, high $T \overset{?}{<} T_c$

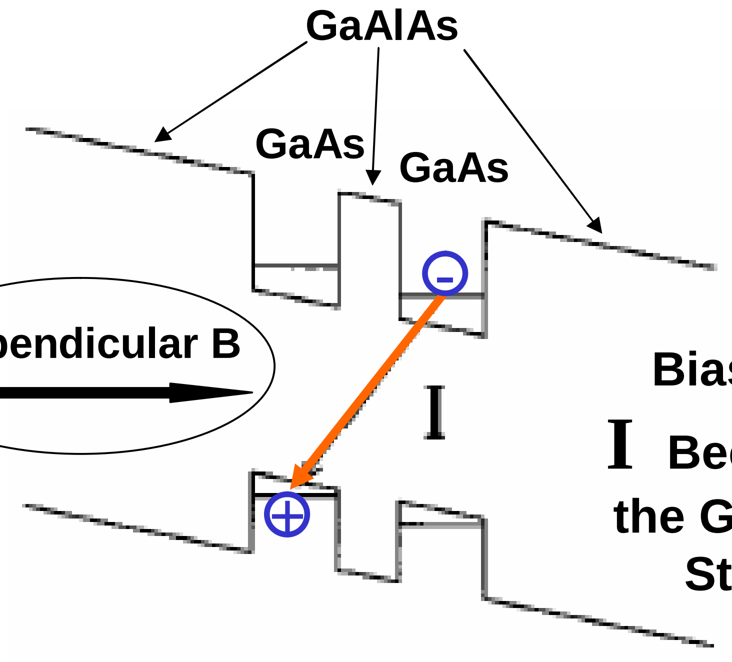
at high densities n:

§ internal **X** structure: overlap of Fermions **X's are not true Bosons!?**

§ screening of the e-h interaction **transition to the e-h liquid?**

Keldysh 1968 **Halperin & Rice 1968**

Indirect Excitons in Quantum Wells



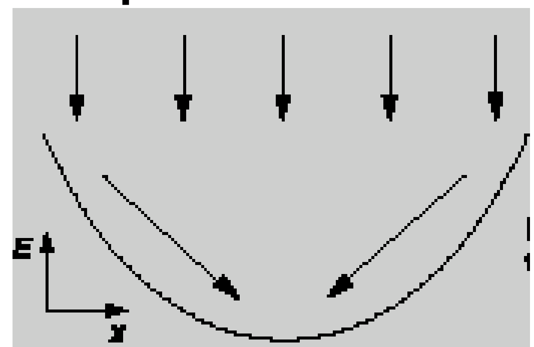
Perpendicular B

Bias V_g:

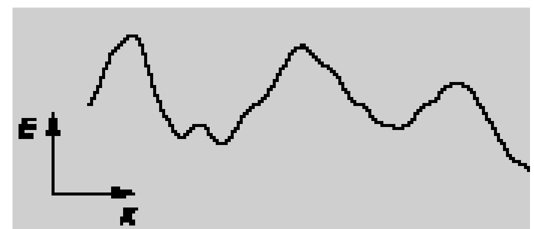
I Becomes the Ground State

Spatially-indirect exciton (I)

X photoexcitation



X relaxation to trap bottom



In-plane random potential

Radiative Lifetimes are Long:

~ e-h overlap in real space

- **Increases Binding Energies**
- **Changes Critical Behavior**

Increased T_c is expected

Fukuzawa et al. 1990

Kono et al. 1995

Butov et al. 1994, 2002

Increase of T_c in Strong Magnetic Fields

$$T_c = E_0 \frac{1-2n}{2\ln(n^{-1}-1)}$$

Kuramoto & Horie 1978   Lerner & Lozovik 1978

Landau Level Filling Factor $n = 2p l_B^{\,2} n$

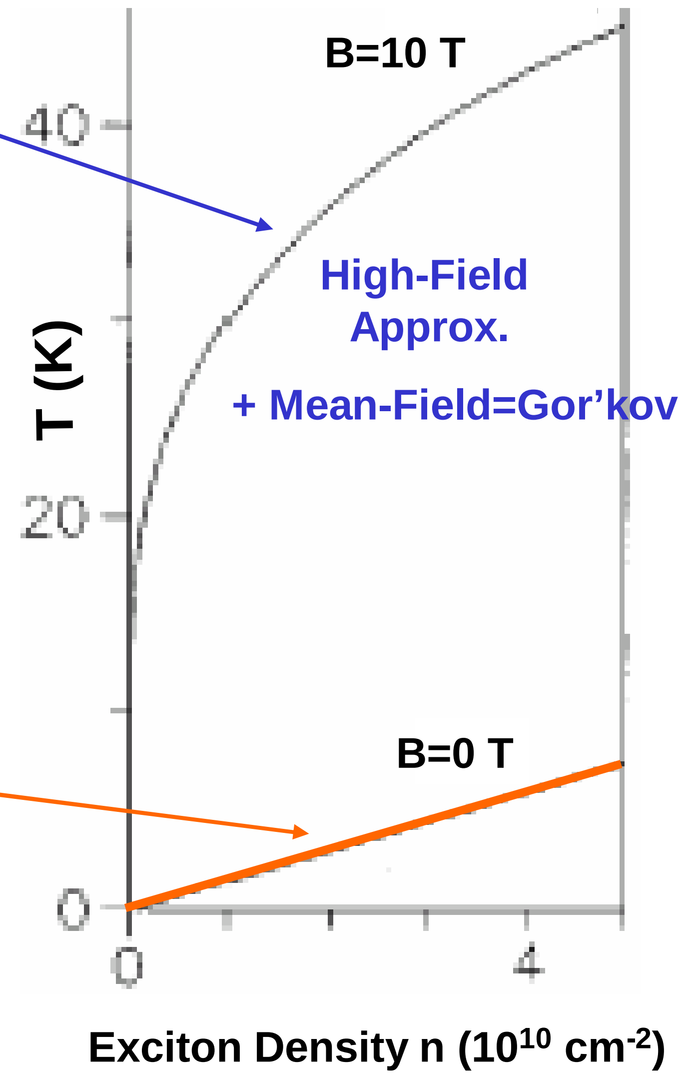
2D Magnetoexciton binding energy $E_0 = \sqrt{\frac{p}{2}}\,\frac{e^2}{e l_B} \propto \sqrt{B}$

Magnetic Length $l_B = \left(\mathbf{h}c/eB\right)^{1/2}$

Characteristic 2D Temperature

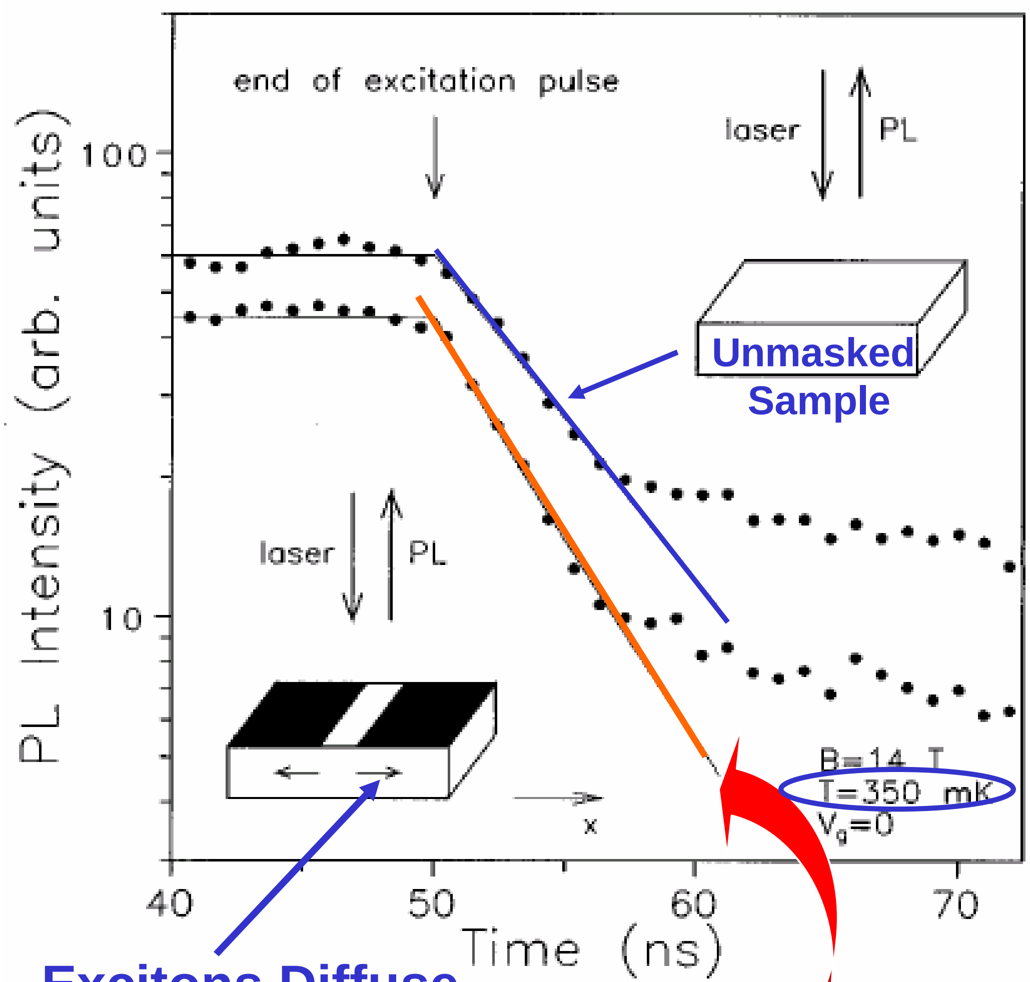
$$T_0 = \frac{2p\mathbf{h}^2 n}{m k_B} \propto n$$

Excitons treated as 2D Ideal Bosons



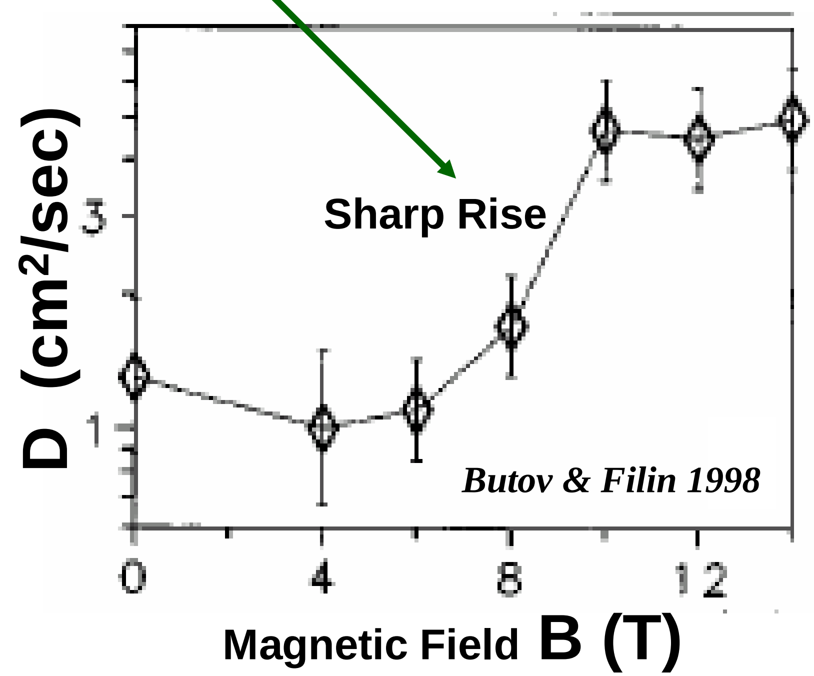
GaAs QW

Low-T Exciton Magneto-Transport



Excitons Diffuse Under the Mask

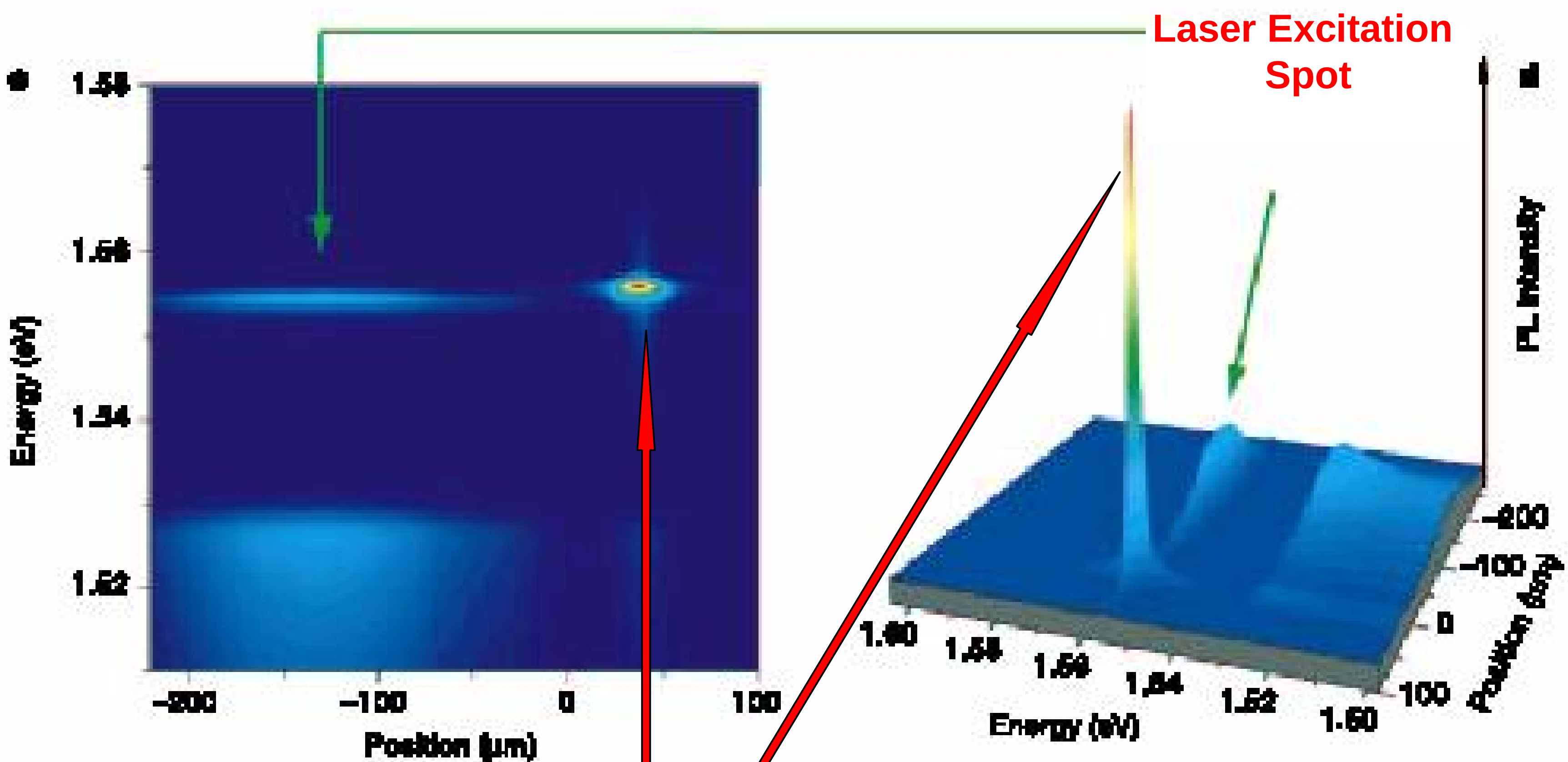
classical X magneto-transport?
weak localization suppresion in B?
No way to explain *ABD & Bauer 1995* *Arseev & ABD 1998*



Exciton Diffusion Constant D can be deduced

The apparent PL decay rate is higher

Towards BEC of excitons in potential traps



Huge local enhancement of PL:

Cold indirect excitons collect in the trap

T = 1.6 K **n ~ 10^{11} cm^{-2}**

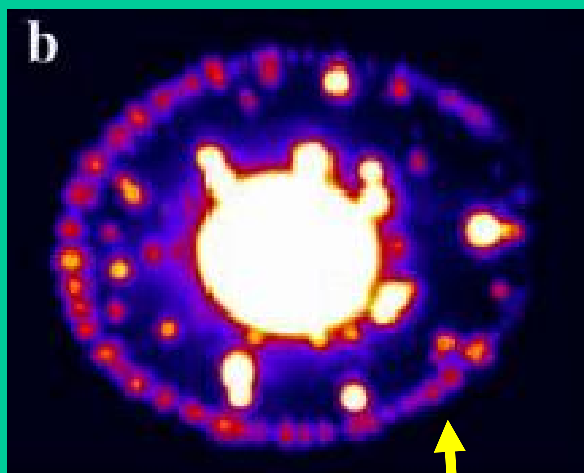
Butov et al. Nature **417**, 47 (2002)

VOLUME 92, NUMBER 11 PHYSICAL REVIEW LETTERS week ending 19 MARCH 2004

Formation Mechanism and Low-Temperature Instability of Exciton Rings

L. V. Butov,[1,2] L. S. Levitov,[3] A. V. Mintsev,[1] B. D. Simons,[4] A. C. Gossard,[5] and D. S. Chemla[1,6]
[1]*Materials Sciences Division, E. O. Lawrence Berkeley National Laboratory, Berkeley, California 94720, USA*
[2]*Department of Physics, University of California San Diego, La Jolla, California 92093-0319, USA*

Exciton Rings



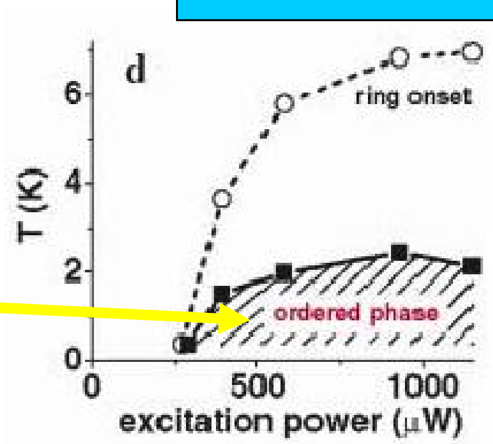
T = 380 mK

Striking spatial photoluminescence patterns: rings, bright spots

- **Span macroscopic scales**
- **Explained classically**

Butov et al. Nature **418**, 751 (2003)

Snoke et al. Nature **418**, 754 (2003)



Regularly spaced beads formed by *cold* excitons

Quantum degeneracy reached?

Composite Bosons Condense

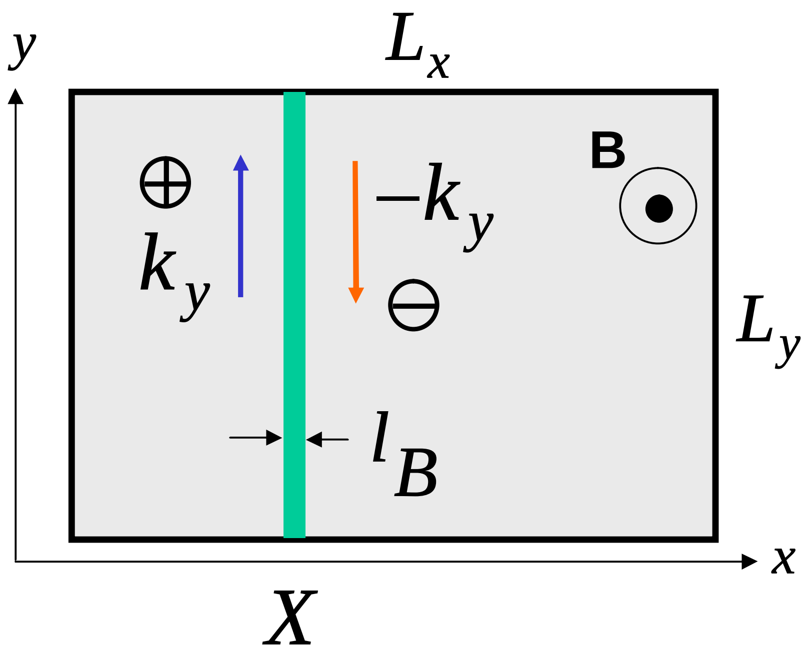
(Exactly-Solvable?)

Pairing Many-Body Models

T = 0

Electrons (e) and Holes (h) In Zero LL

$$j_{hn=0X}(\mathbf{r}) = j^{*}_{en=0X}(\mathbf{r}) = \exp(iXy/l_B^{2})\exp\left[-\frac{(x-X)^{2}}{4l_B^{2}}\right]$$



Landau gauge $\mathbf{A} = (0, Bx, 0)$

Magnetic length $l_B = \left(\frac{\mathbf{h}c}{eB}\right)^{1/2}$ $l_B = 81$Å $B = 10\text{T}$

X as a 1D momentum:

$$X = \pm k_y l_\mathrm{B}^{2} \quad \oplus \quad \ominus$$

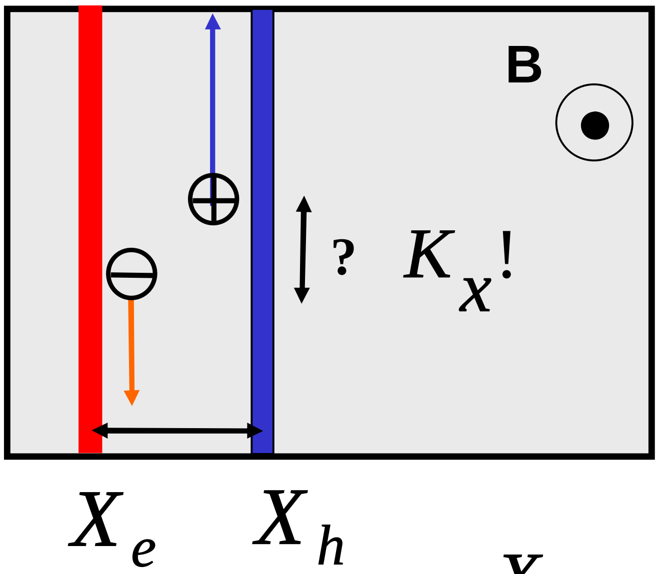
Macroscopic Degeneracy in X

$$N_0 = \frac{L_x L_y}{2p l_B^{2}} \propto B$$

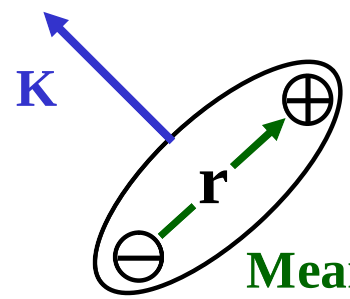
Kinetic Energy is Quenched

$$\frac{1}{2}\mathbf{h}w_{ce(h)} = \frac{1}{2}\mathbf{h}\frac{eB}{m_{e(h)}c}$$

2D e-h Pair in Zero LL: A Magnetoexciton (MX)



MX: a neutral composite Boson



CM momentum

$$\mathbf{K} = (K_x, K_y)$$

Mean dipole moment e**r**

$$\mathbf{r} = \langle \mathbf{r}_h - \mathbf{r}_e \rangle = \mathbf{K} \times \hat{\mathbf{z}} l_B^2$$

$$X_h - X_e = K_y l_B^2$$

$$k_{hy} + k_{ey}$$

Gor'kov & Dzyaloshinskii 1967

2D MX annihilation operator

$$Q_{\mathbf{K}} = \frac{1}{\sqrt{N_0}} \sum_X \exp(iK_x X)\, \hat{a}_{X - \frac{1}{2}K_y} \hat{b}_{X + \frac{1}{2}K_y}$$

⊖ ⊕ $(l_B = 1)$

Lerner & Lozovik 1980

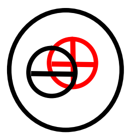
ABD& Lozovik 1983

Callin & Halperin 1984

K=0 Magnetoexciton

CM momentum

$$\mathbf{K} = (K_x, K_y) = 0$$



Mean dipole moment

$$\mathbf{r} = \langle \mathbf{r}_h - \mathbf{r}_e \rangle = \mathbf{K} \times \hat{\mathbf{z}} l_B{}^2 = 0$$

$$Q_0 = \frac{1}{\sqrt{N_0}} \sum_X \hat{a}_X \hat{b}_X$$

Extended CM motion

Bound relative *e-h* motion

Binding Energy

$$E_0 = \sqrt{\frac{p}{2}} \frac{e^2}{e l_B} \propto \sqrt{B}$$

$$E_0 \sim 10\,\text{meV}$$

GaAs $B \sim 10\text{T}$

MX as a Composite Boson

e h X

Fermion + Fermion = Boson?

$$[Q_{\mathbf{K}'}, Q_{\mathbf{K}}] = 0 \qquad [Q^{+}_{\mathbf{K}'}, Q^{+}_{\mathbf{K}}] = 0$$ ü

[A,B]=AB-BA ***commutator***

Not exactly:

$$[Q_{\mathbf{K}'}, Q^{+}_{\mathbf{K}}] = d_{\mathbf{K}'\mathbf{K}} + \hat{F}(\mathbf{K}', \mathbf{K})$$ ~ relative occupancy

Keldysh & Kozlov 1968

MX: $$[Q_0, Q_0] = 1 - \frac{\hat{N}_e + \hat{N}_h}{N_0}$$

Particle # operators $\hat{N}^{+}_{e}$ $\hat{N}^{+}_{h}$

Can Composite Bosons occupy the same state? Yes!

Composite Fermions

Fermion + Boson = Fermion?

hypothetically: e h

{A,B}=AB+BA ***anticommutator***

Not exactly: $\{Q_{\mathbf{K}'}, Q^{+}_{\mathbf{K}}\} = d_{\mathbf{K}'\mathbf{K}} + \hat{G}(\mathbf{K}', \mathbf{K})$ ~ relative occupancy

$$\{Q_0, Q_0\} = 1 - \frac{\hat{N}_e - \hat{N}_h}{N_0}$$

But we should not care too much: ü $\{Q_{\mathbf{K}'}, Q_{\mathbf{K}}\} = 0$ $\{Q^{+}_{\mathbf{K}'}, Q^{+}_{\mathbf{K}}\} = 0$

$\left(Q_{\mathbf{K}}^{+}\right)^2 = 0 \Longrightarrow$ ***Composite Fermions cannot occupy the same state!***

Interaction Hamiltonian in Strong B

$$H_{\text{int}} = \frac{1}{2}\int d\mathbf{r}_1 \int d\mathbf{r}_2\, U_{ee}(\mathbf{r}_1 - \mathbf{r}_2)\hat{\Psi}_e{}^{+}(\mathbf{r}_1)\hat{\Psi}_e{}^{+}(\mathbf{r}_2)\hat{\Psi}_e(\mathbf{r}_2)\hat{\Psi}_e(\mathbf{r}_1)$$

$$+\frac{1}{2}\int d\mathbf{r}_1 \int d\mathbf{r}_2\, U_{hh}(\mathbf{r}_1 - \mathbf{r}_2)\hat{\Psi}_h{}^{+}(\mathbf{r}_1)\hat{\Psi}_h{}^{+}(\mathbf{r}_2)\hat{\Psi}_h(\mathbf{r}_2)\hat{\Psi}_h(\mathbf{r}_1)$$

$$+\int d\mathbf{r}_1 \int d\mathbf{r}_2\, U_{eh}(\mathbf{r}_1 - \mathbf{r}_2)\hat{\Psi}_e{}^{+}(\mathbf{r}_1)\hat{\Psi}_h{}^{+}(\mathbf{r}_2)\hat{\Psi}_h(\mathbf{r}_2)\hat{\Psi}_e(\mathbf{r}_1)$$

Like in FQHE:

Poject onto zero Landau levels (LLs)

$$\hat{\Psi}_e(\mathbf{r}) = \sum_{n=0}^{\infty}\sum_{X} j_{enX}(\mathbf{r})\hat{a}_{nX} \;\; \mathbf{a} \;\; \sum_{X} j_{eX}(\mathbf{r})\hat{a}_{X}$$

$$n = 0$$

$$\hat{\Psi}_h(\mathbf{r}) = \sum_{n=0}^{\infty}\sum_{X} j_{hnX}(\mathbf{r})\hat{b}_{nX} \;\; \mathbf{a} \;\; \sum_{X} j_{hX}(\mathbf{r})\hat{b}_{X}$$

And forget about the absent kinetic energy ...

Two-Component e-h System in Zero LL

One-component electron sytem: FQHE, Wigner (electron) crystal, ...

$$H_{\rm int} = \frac{1}{2}\sum U_{ee}(X_1, X_2; X_2', X_1')\, \hat{a}_{X_1}^{+}\, \hat{a}_{X_2}^{+} \hat{a}_{X_2'} \hat{a}_{X_1'}$$

$$+\frac{1}{2}\sum U_{hh}(X_1, X_2; X_2', X_1')\, \hat{b}_{X_1}^{+} \hat{b}_{X_2}^{+} \hat{b}_{X_2'} \hat{b}_{X_1'}$$

$$+\sum U_{eh}(X_1, X_2; X_2', X_1')\, \hat{a}_{X_1}^{+} \hat{b}_{X_2}^{+} \hat{b}_{X_2'} \hat{a}_{X_1'}$$

Symmetric Two-component Systems

$$U_{ee}(\mathbf{r}) = U_{hh}(\mathbf{r}) = -U_{eh}(\mathbf{r})$$ interactions

$$j_{hX}(\mathbf{r}) = j^{*}_{eX}(\mathbf{r})$$ time-reversal counterparts

$$E_h(X) + E_e(X) = \text{const}$$ single-particle energies

only this is a bit exotic!

BEC as an exact state

Lerner& Lozovik 1981
ABD& Lozovik 1983, 1991
Rice et al. 1985

MX Condensate

Exact Quantum Equation of Motion

$$[H_{\text{int}}, {Q_0}^{+}] = (-E_0)\,{Q_0}^{+}$$

"Hidden Symmetry"

Non-Interacting Gas of Composite Particles

$$\forall \;\; N_e = N_h = N \le N_0$$

Exact Many-Body State: *K*=0 MX Condensate

$$\left({Q_0}^{+}\right)^{N} |0\rangle \equiv |N\rangle \qquad H_{\text{int}}\,|N\rangle = N(-E_0)\,|N\rangle$$

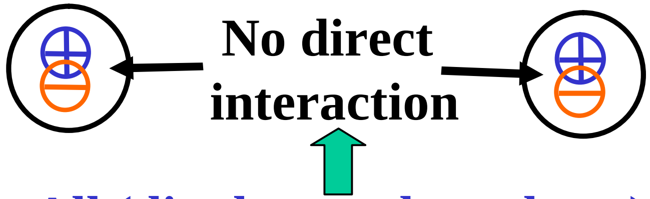
The Ground State of the Many-Body System?

We think so.

But no strict theorems known.

Ideal Gas? How to Understand This?

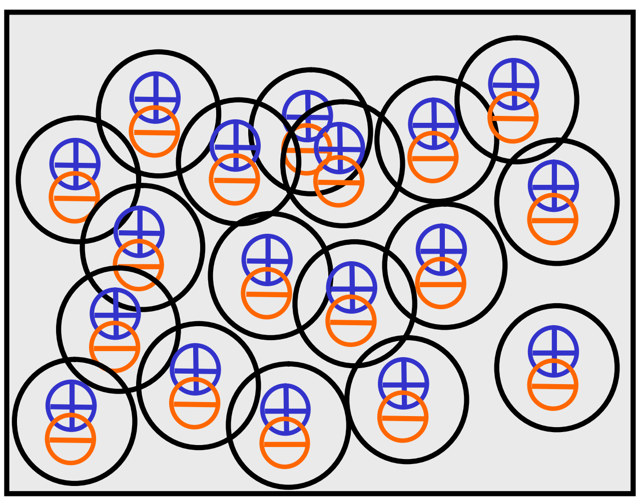
A partial explanation



K=0 MXs: **All (dipole, quadrupole, ...) moments = 0**

§Polarization (Van der Waals) interactions?

Holds for High-Densities



Overlapping e-h pairs

= Overlapping Fermions

Even for e- and h- filling factors

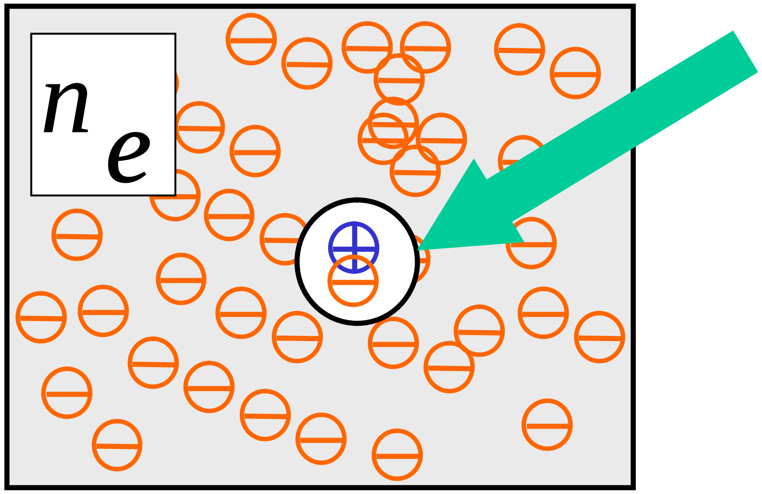
$$n_e = \frac{N_e}{N_0} \to 1 \qquad n_h = \frac{N_h}{N_0} \to 1$$

§ Screening??

§Exchange interaction???

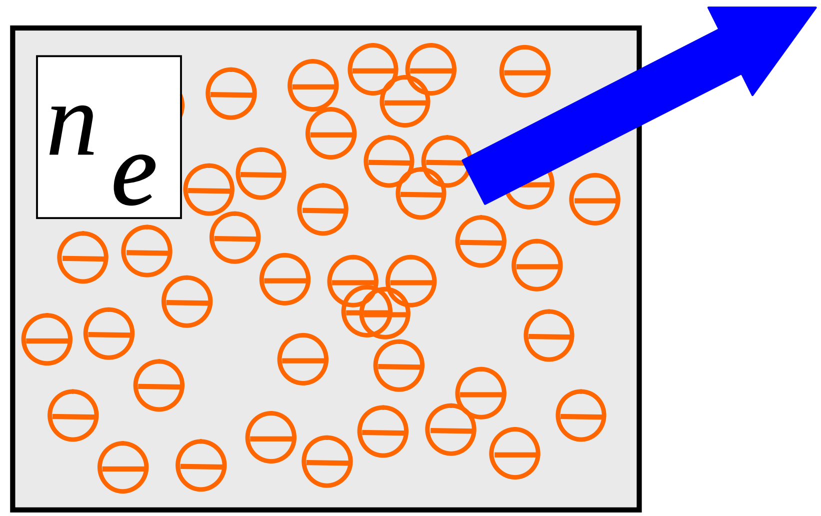
Appear to be absent

Magneto-Optics of Interacting 2D Electrons



I. Photo-excited e-h pair

II. Photorecombination



Carries information about e-e correlations (?)

Luminescence Operator

$$\hat{L}_{PL} = p_{cv}\int d\mathbf{r}\,\hat{\Psi}_e(\mathbf{r})\hat{\Psi}_h(\mathbf{r}) \;\; \mathbf{a} \;\; p_{cv}Q_0$$

K=0 MX does not interact with anything

In symmetric QWs in high fields:

Recombination Energy= $E_{\text{gap}} - E_0$

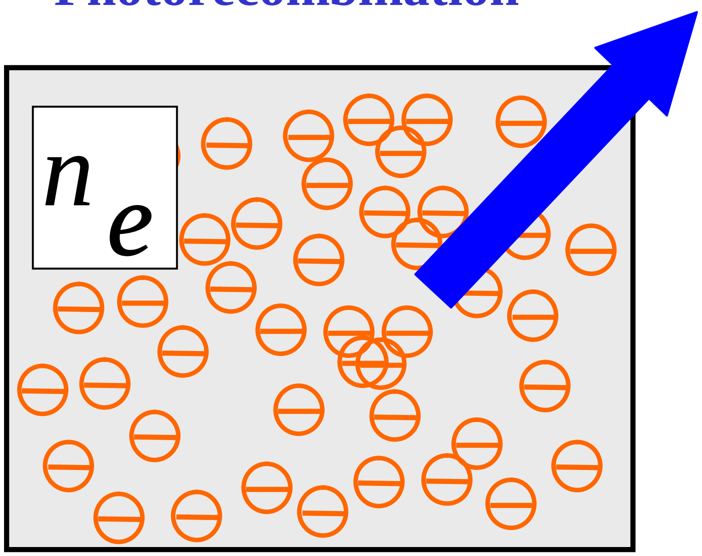
MacDonald et al. 1992

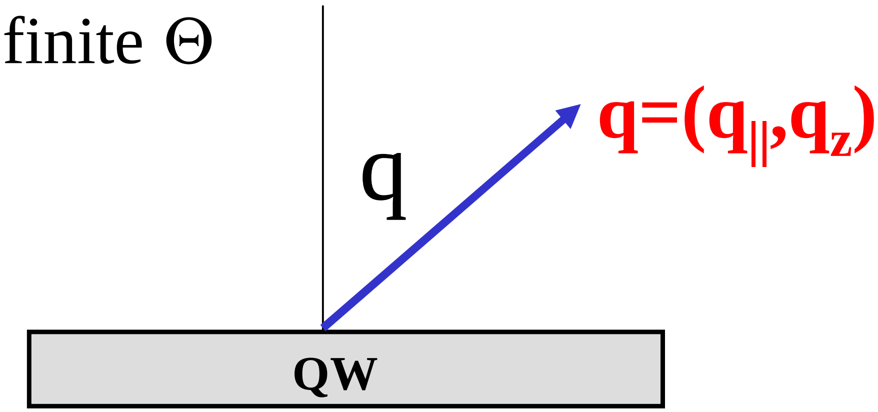
Apalkov and Rashba 1992

Chen and Quinn 1994

Magneto-Optical Probing of Electrons

Photorecombination





$\Theta \neq 0$: **Carries information about e-e correlations**

Luminescence Operator

$$\hat{L}_{PL} = p_{cv}\int d\mathbf{r}\,\exp(i\mathbf{q}_{\|}\cdot\mathbf{r})\hat{\Psi}_e(\mathbf{r})\hat{\Psi}_h(\mathbf{r}) \;\;\mathbf{a}\;\; p_{cv}Q_{\mathbf{q}_{\|}}$$

$\mathbf{q}_{\|}$- MX probes the electron system

Recombination Energy (q) **=** $E_{\text{gap}} + E(\text{collective effects})$

Yet More Exact States: Charged Systems

$$N_e \neq N_h$$

e (or any # of e or h)

One Overcondensate Electron

$$a_X^{+}\left(Q_0^{+}\right)^N |0\rangle$$

does not polarize the condensate!

$$\frac{N_e - N_h}{N_0} = \frac{1}{3}, \frac{1}{5}, \ldots \frac{p}{q}$$

remaining electrons

$N_h = N$ ***holes***

$+N$ ***electrons***

FQHE over the condensate

MacDonald & Rezayi 1990

$$(\text{Laughlin State})^{+}\left(Q_0^{+}\right)^N |0\rangle$$

Only later it became clear: **Exact but NOT the Ground States**

2D Charged Triplet MX X^{-} Formed!

Hidden Symmetry

$$H = H_{\text{int}} - (m_e \hat{N}_e + m_h \hat{N}_h)$$

Unitary Transformation

$$S = \exp(f\hat{L})$$

$$H \to \tilde{H} = SHS^{+} = H$$

$$\hat{L} = Q_0^{+} - Q_0$$

anti-Hermitian generator

$$m_e + m_h = -E_0$$

$$[H, \hat{L}] = 0 \longrightarrow [H_{\text{int}}, Q_0^{+}] = -E_0 Q_0^{+}$$

$S^{+} |0\rangle$ **BCS state: the coherent state of composite particles** Q_0^{+}

Exact in this model!

Hidden Symmetry Explained

$$H = H_{\text{int}} - (\mu_e \hat{N}_e + \mu_h \hat{N}_h)$$

Unitary Transformation

$$H \rightarrow \tilde{H} = SHS^{+} \qquad S = \exp(f\hat{L}) \qquad \hat{L} = Q_0^{+} - Q_0$$

$$Q_0 = \sum_X b_X a_X$$ **annihilation operator of the composite Boson**

Generates (special!) Bogoliubov Rotations in the Isospin Space of Two Components:

$$a_X \rightarrow \tilde{a}_X = S a_X S^{+} = \cos f \, a_X - \sin f \, b_X^{+}$$

$$b_X \rightarrow \tilde{b}_X = S b_X S^{+} = \cos f \, b_X + \sin f \, a_X^{+}$$

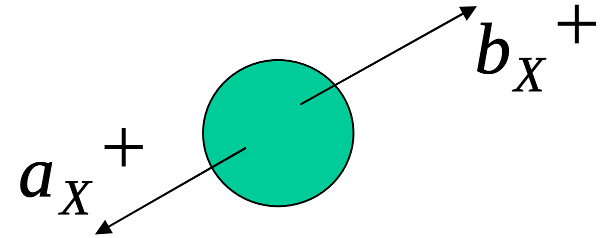
angle of rotation f does not depend on quantum # X (momentum)

Creation of e-h Pairs From Vacuum

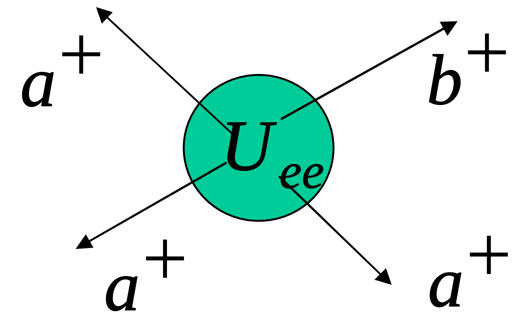
$$a_X^+ a_X \to S a_X^+ a_X S^+$$

$$= -(\sin f \cos f) a_X^+ b_X^+ + \ldots$$

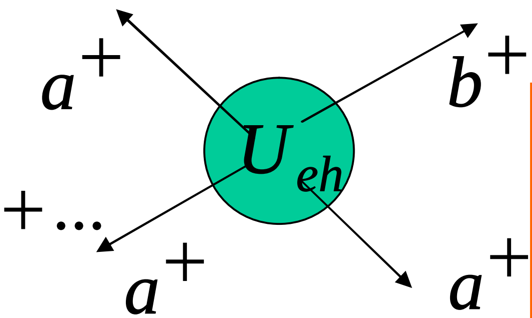
cancel by the choice m



$$U_{ee} a^+ a^+ a a \to U_{ee} a^+ a^+ b^+ b^+ + \ldots$$



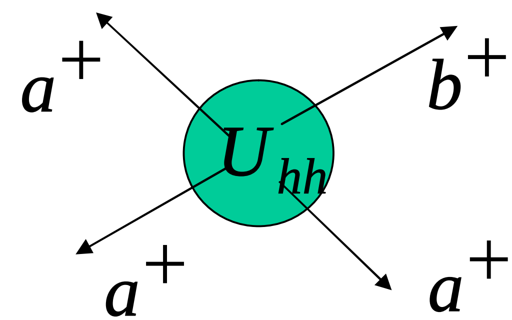
$$U_{eh} a^+ b^+ b a \to U_{eh} a^+ b^+ a^+ b^+ + \ldots$$



$U_{ee} = U_{hh} = -U_{eh}$

cancel automatically!

$$U_{hh} a^+ a^+ a a \to U_{hh} a^+ a^+ b^+ b^+ + \ldots$$



Statistics, Generators, and Lie Algebras

$$Q = \sum_{X}^{N_0} \hat{a}_X \hat{b}_X$$

Fermion a + Fermion b = Composite Boson Q

Boson a + Boson b = Composite Boson Q

$$[Q, Q^{+}] = N_0 \, \mathbf{m} \, \hat{N}$$

$$\hat{N} = \hat{N}_a + \hat{N}_b$$

Particle # operators

Generators $K_0 = \frac{1}{2}\left(\hat{N} - N_0\right)$ $K^{-} = Q$ $K^{+} = Q^{+}$

Lie Algebra

$$[K_0, K^{-}] = -K^{-}$$

$$[K_0, K^{+}] = K^{+}$$

$$[K^{-}, K^{+}] = \mathbf{m}2K_0$$

FF: SU(2)

BB: SU(1,1)

BEC in 2D

Mermin & Wagner 1966

Hohenberg 1967

No macroscopic occupation of a single state → **No LRO**

No condensate!

Superfluidity?

Yes: quasi-condensate $K \approx 0$

Popov 1973

D. Fisher & Hohenberg 1988

Disorder in 2D??

Potential fluctuations in QWs → **"Lakes" of exciton BEC**

Summary

- **Exactly-solvable pairing model:**
  BEC of 2D Magnetoexcitons

- **Angle-Resolved Magneto-Optical Emission from 2D systems**
  a possible probe of collective effects?

$$I(\Theta)$$

- **Trapped quasi-2D (Magneto)-Excitons:**
  ongoing search for BEC of excitons